Foundational Concepts of DFT

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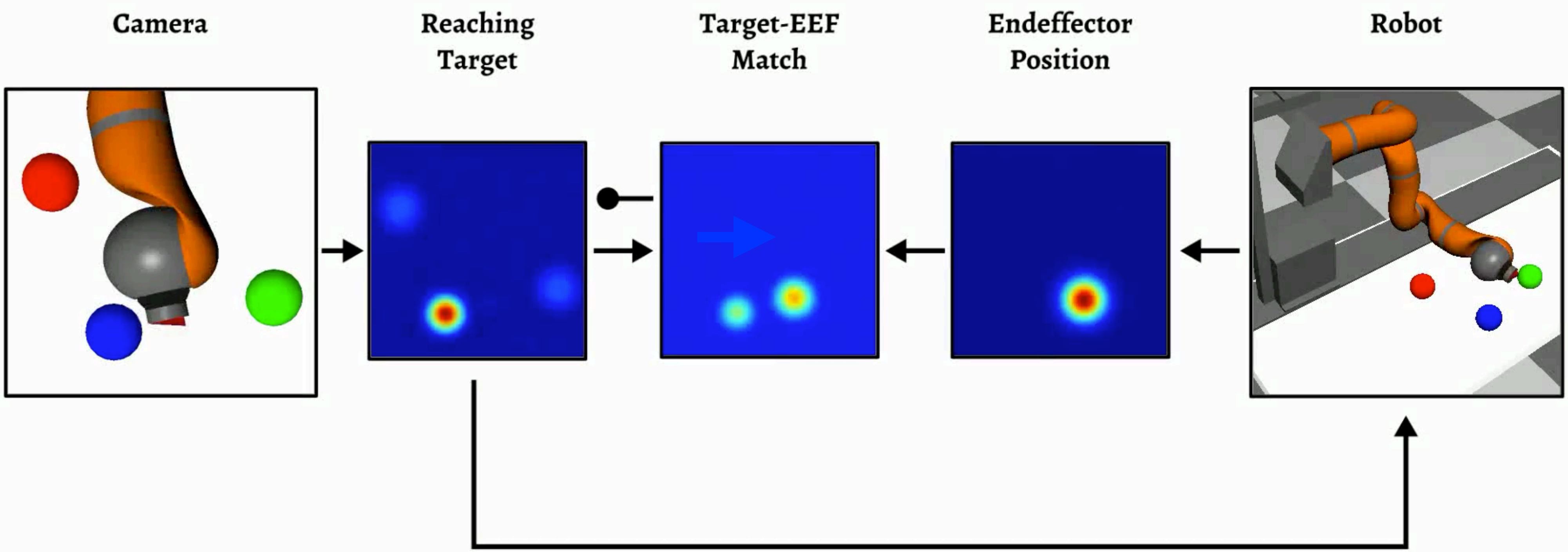
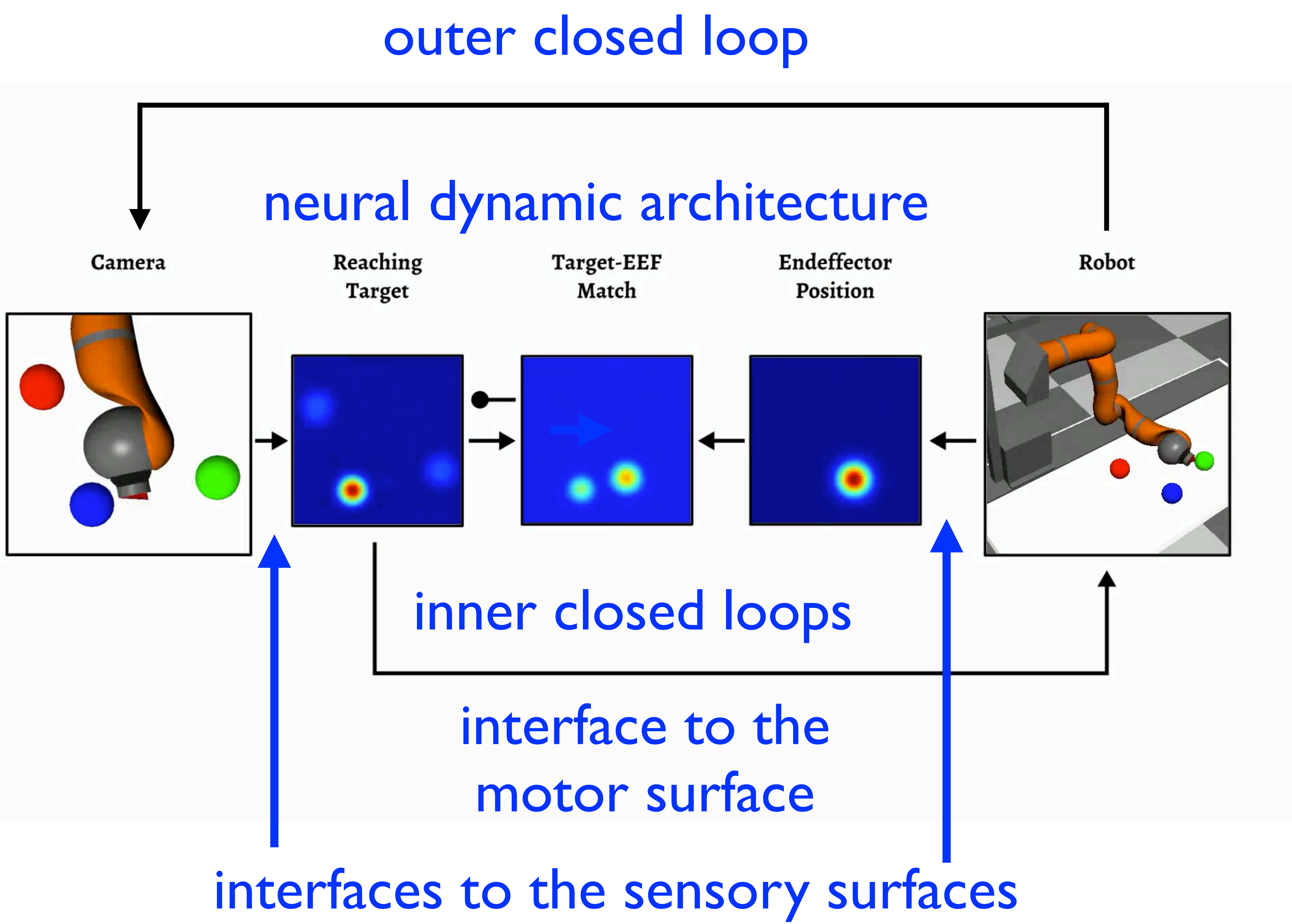
A DFT architecture

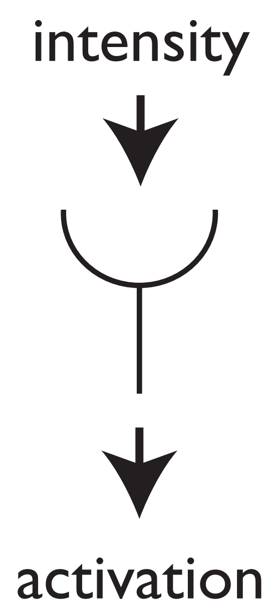


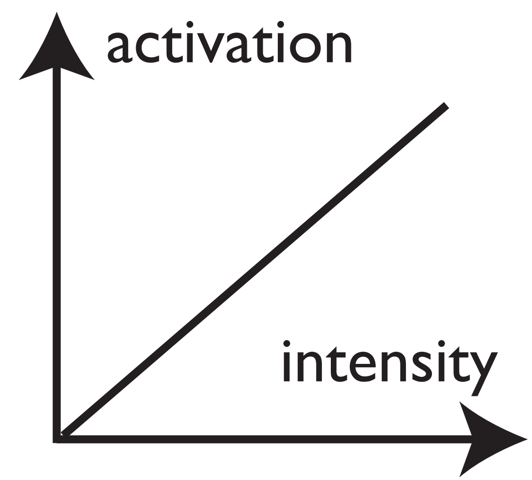


outer closed loop
neural dynamic architecture
Camera
Reaching Target
Target-EEF Match
Endeffector Position
Robot
inner closed loops
interface to the motor surface
interfaces to the sensory surfaces

Sensors

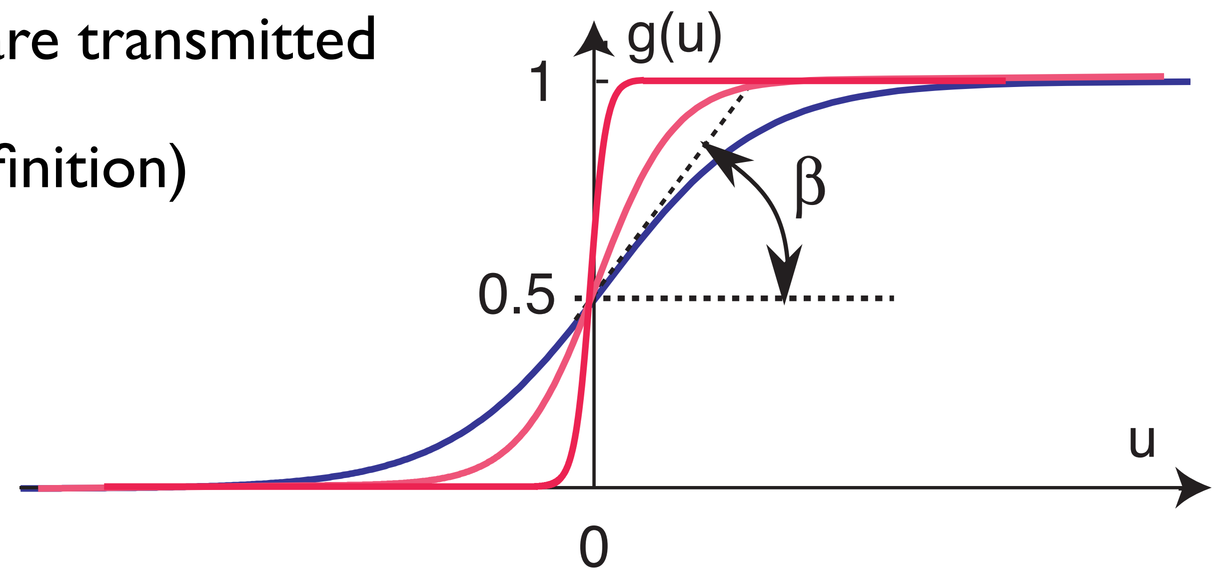
- transform a physical intensity into a neural activation
- intensity: light, sound, displacement
- neural activation: membrane potential, spike rate





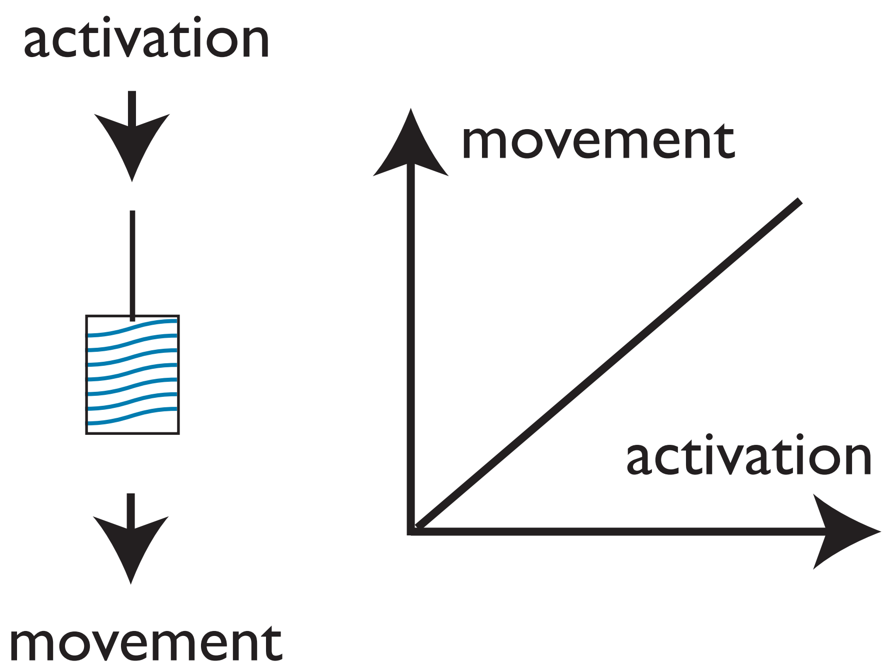
Activation

- activation as an abstraction, defined relative to sigmoidal threshold function
  - low levels of activation are not transmitted (to other neural systems, to motor systems)
  - high levels of activation are transmitted
  - threshold at zero (by definition)



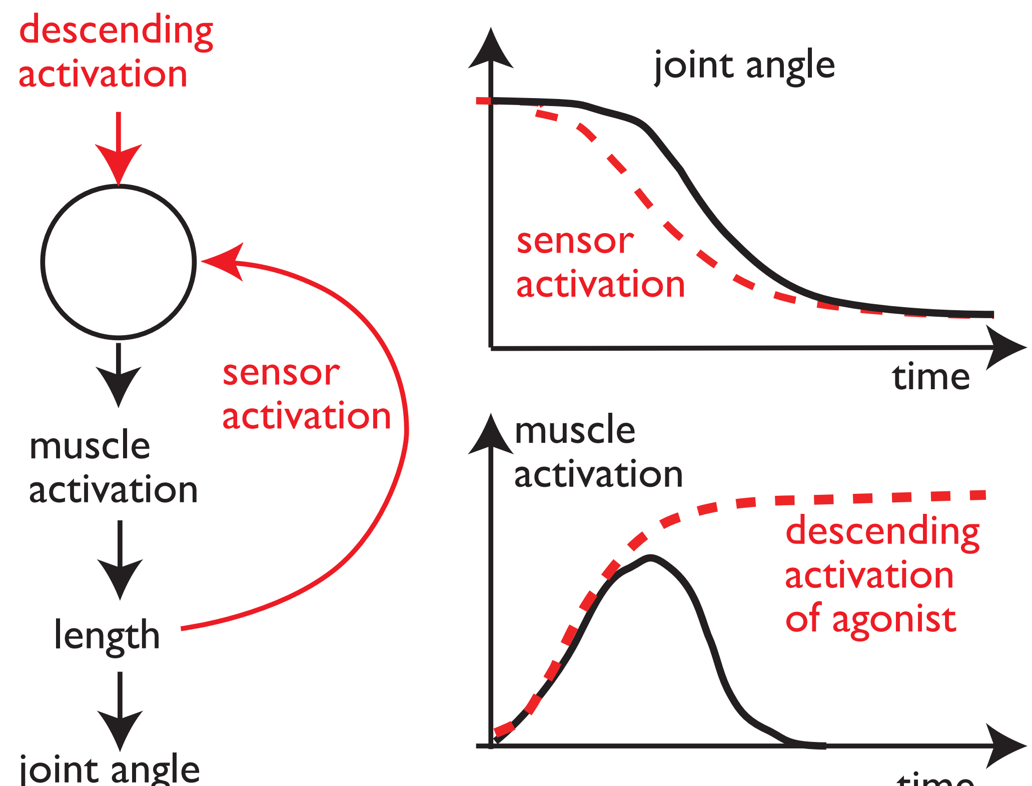
Motors

- transform activation into physical action
- … muscles



Motors

- … actually entails closed loops..
- and is dynamic in nature!



Sensory surfaces

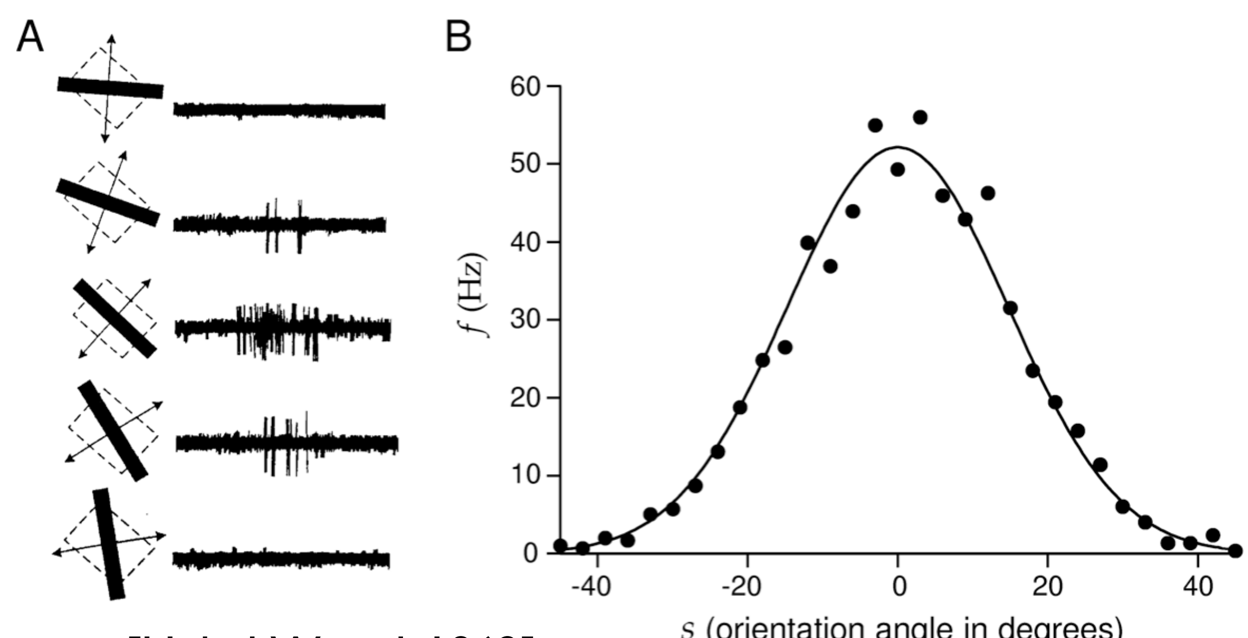
- many sensors…
  - in the retina
  - the cochlea
  - the skin …
- form an anatomical sensory surface…

Functional sensory surfaces

- vision
  - visual space
  - oriented contrasts in visual space
  - movement direction in visual space
- audition
  - pitch
  - formants
- haptics
  - texture ..

Neural representation of functional sensory surfaces

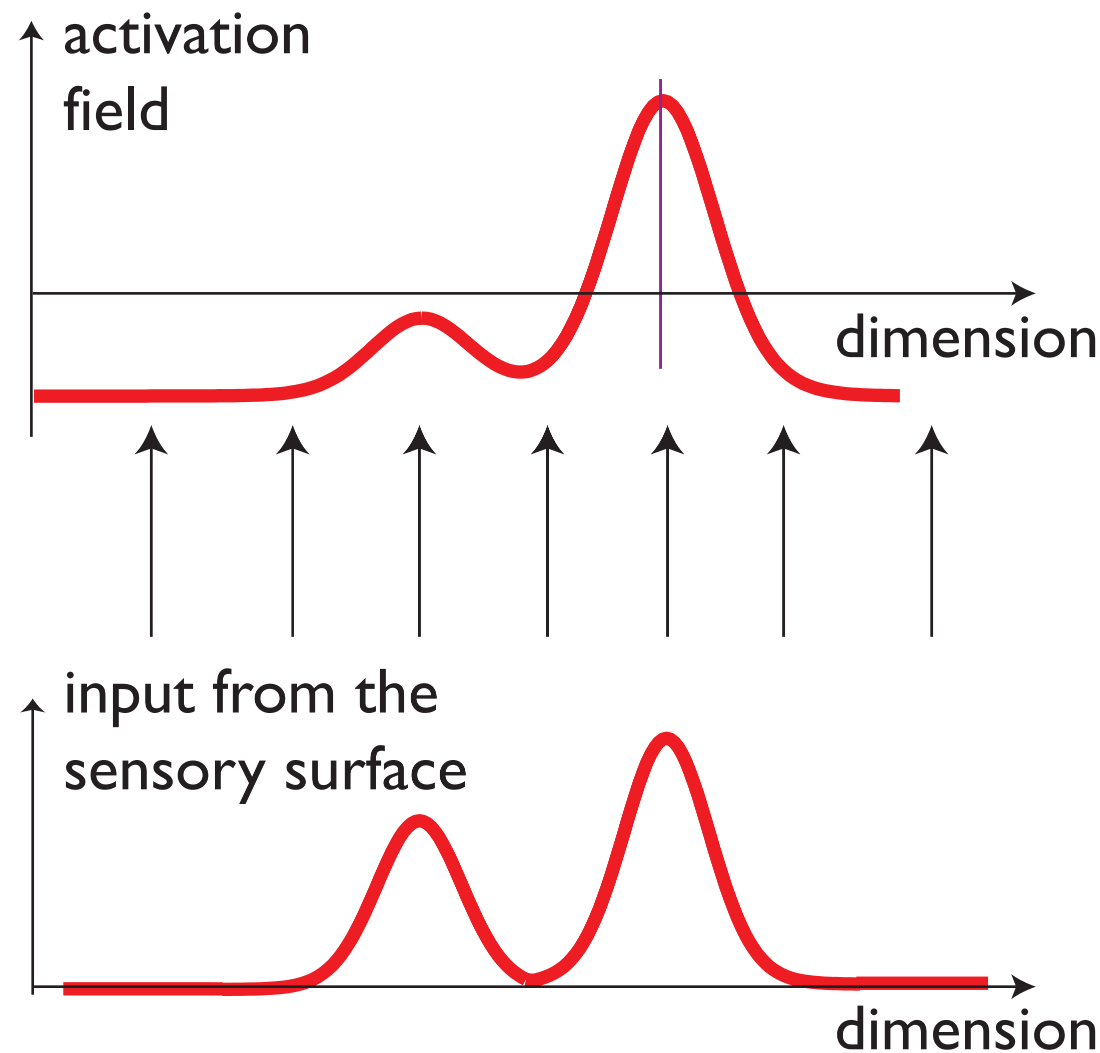
- extract features from the anatomical sensory surfaces by input-driven neural networks (essentially feed-forward)
- as characterized by tuning curves/ receptive fields



[Hubel, Wiesel, 1962]

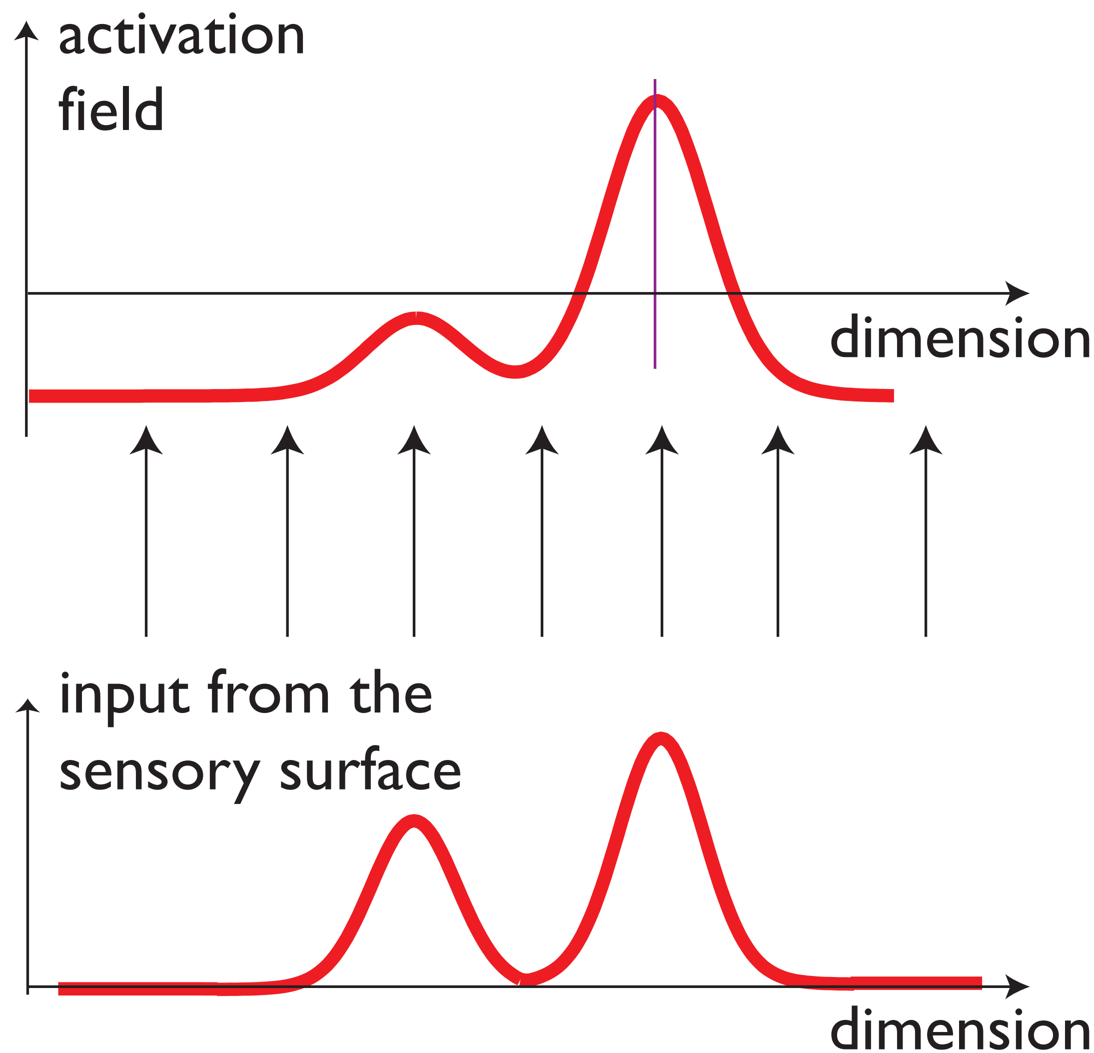
Neural representation of functional sensory surfaces

- leading to neural maps:
- space code/population code

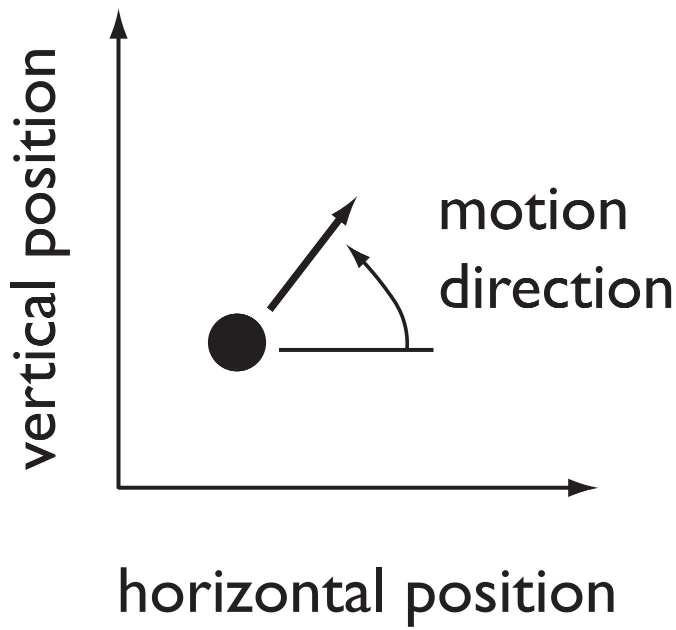


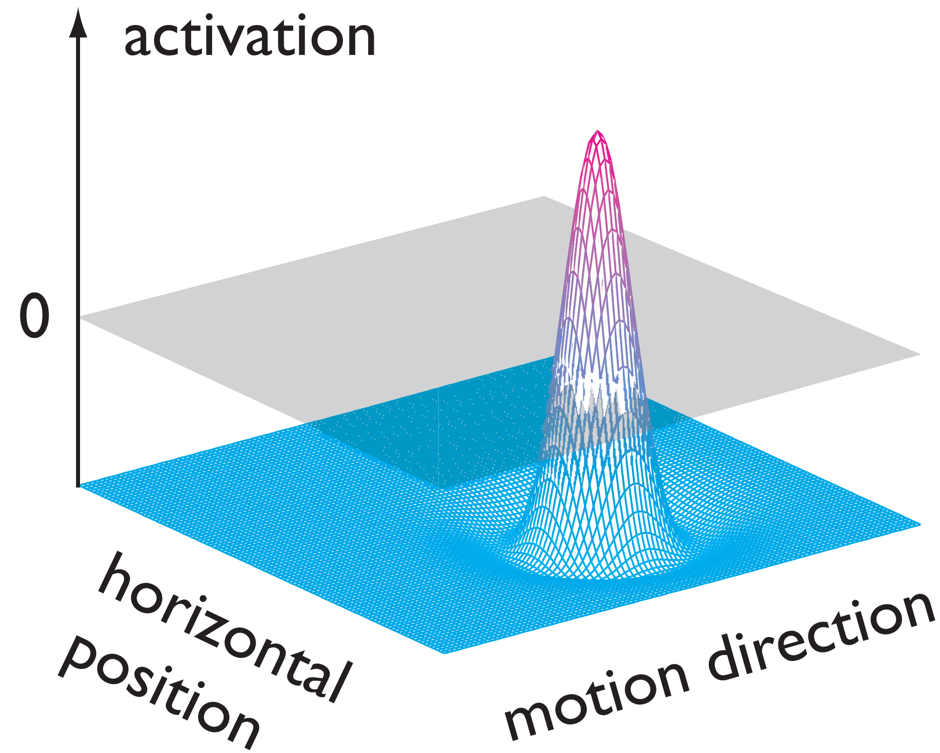
Neural fields

- the discrete sampling of such neural maps by individual neurons does not matter
- => activation fields



Peaks of activation in perceptual neural fields represent objects



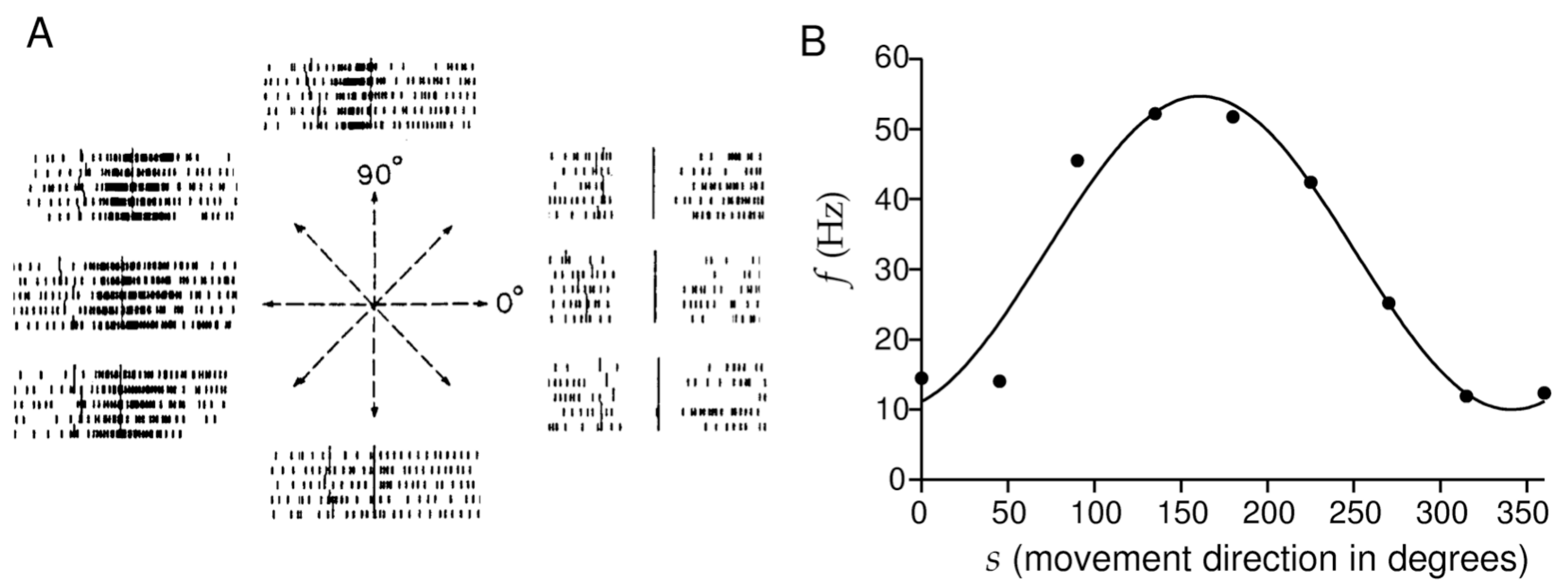


Motor surfaces

- the sets of muscles that actuate the degrees of freedom of the body .. anatomical motor surface

Functional motor surfaces

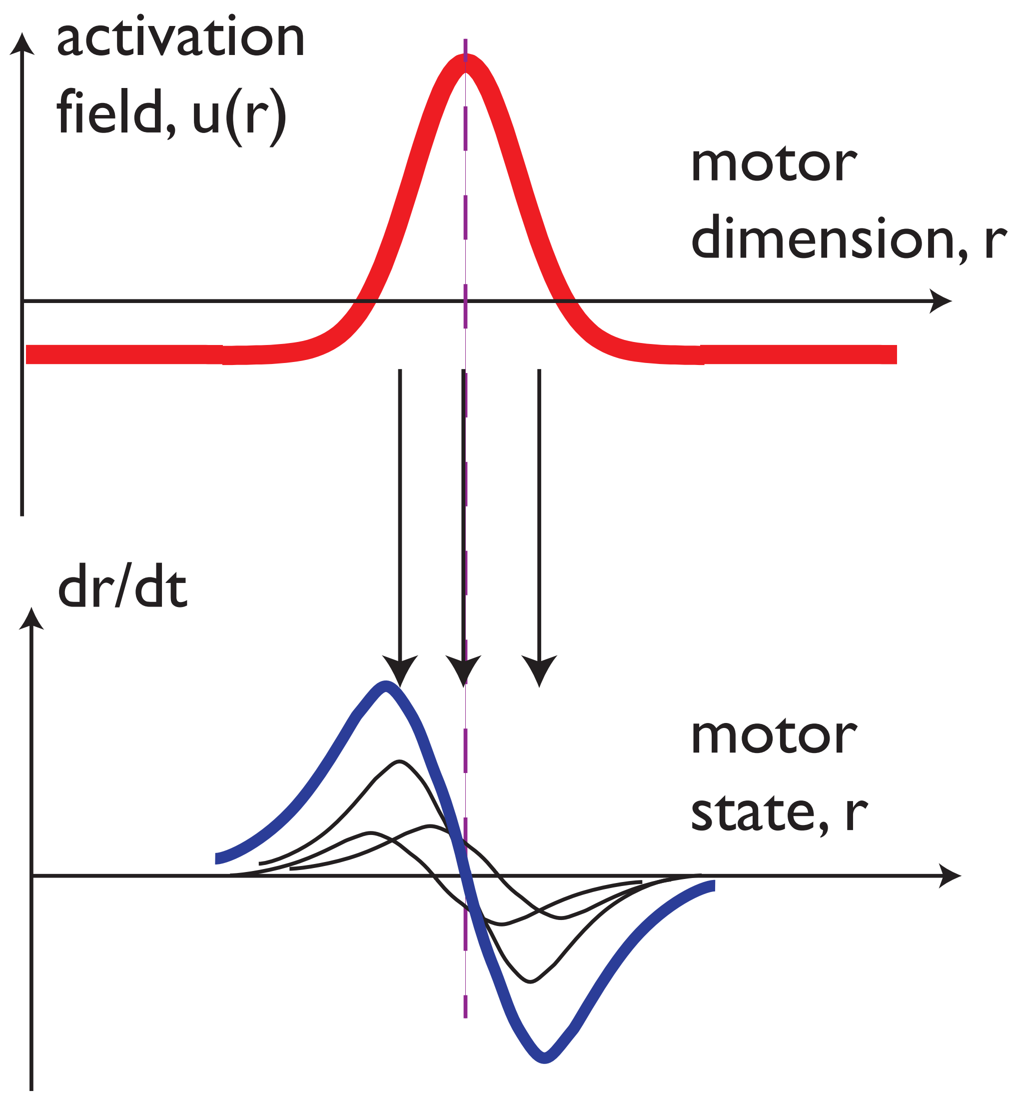
- the parameters describing movements… movement direction, amplitude etc..= functional motor surface
- the (essentially) forward connectivity to the muscular systems (synergies) implement functional motor surfaces..



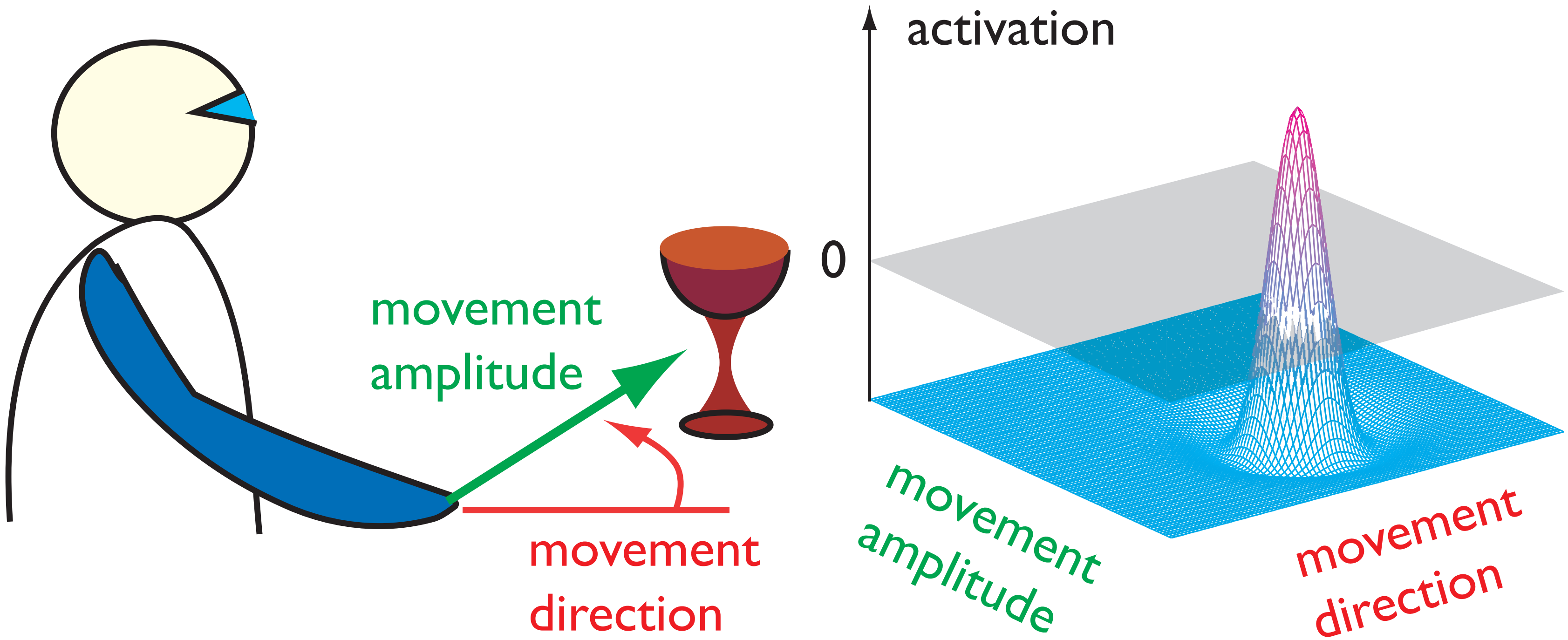
[Georgopoulos, Schwartz, Kalaska, 1986]

Neural fields

- neural fields represent the functional motor surface through their connectivity to the anatomical motor surface

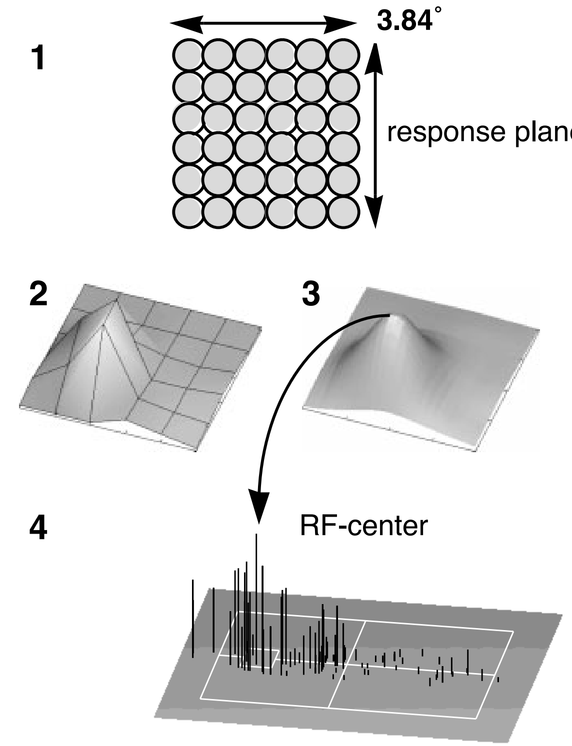


Peaks of activation in motor neural fields represent motor intentions

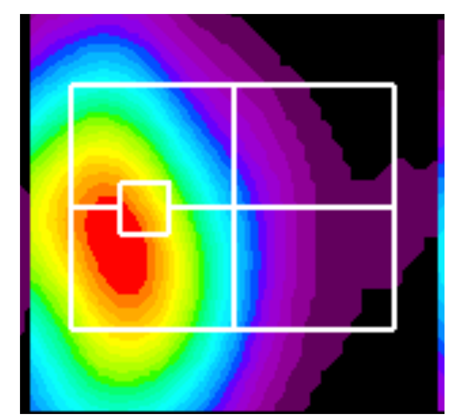


Link to population code

Neural fields: Distributions of population activation (DPA)



- sensory: primary visual cortex
- determine RF profile for each cell
- superpose these weighted by the current neural firing rate
- => DPA defined over retinal space

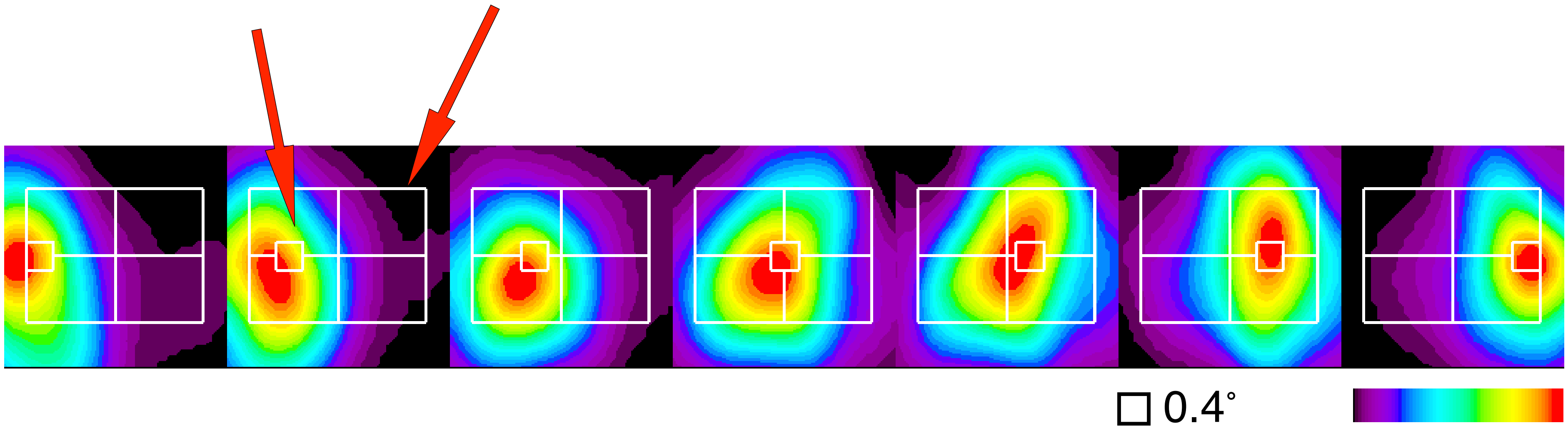


[Jancke et al, J. Neurosci 1999]

DPA

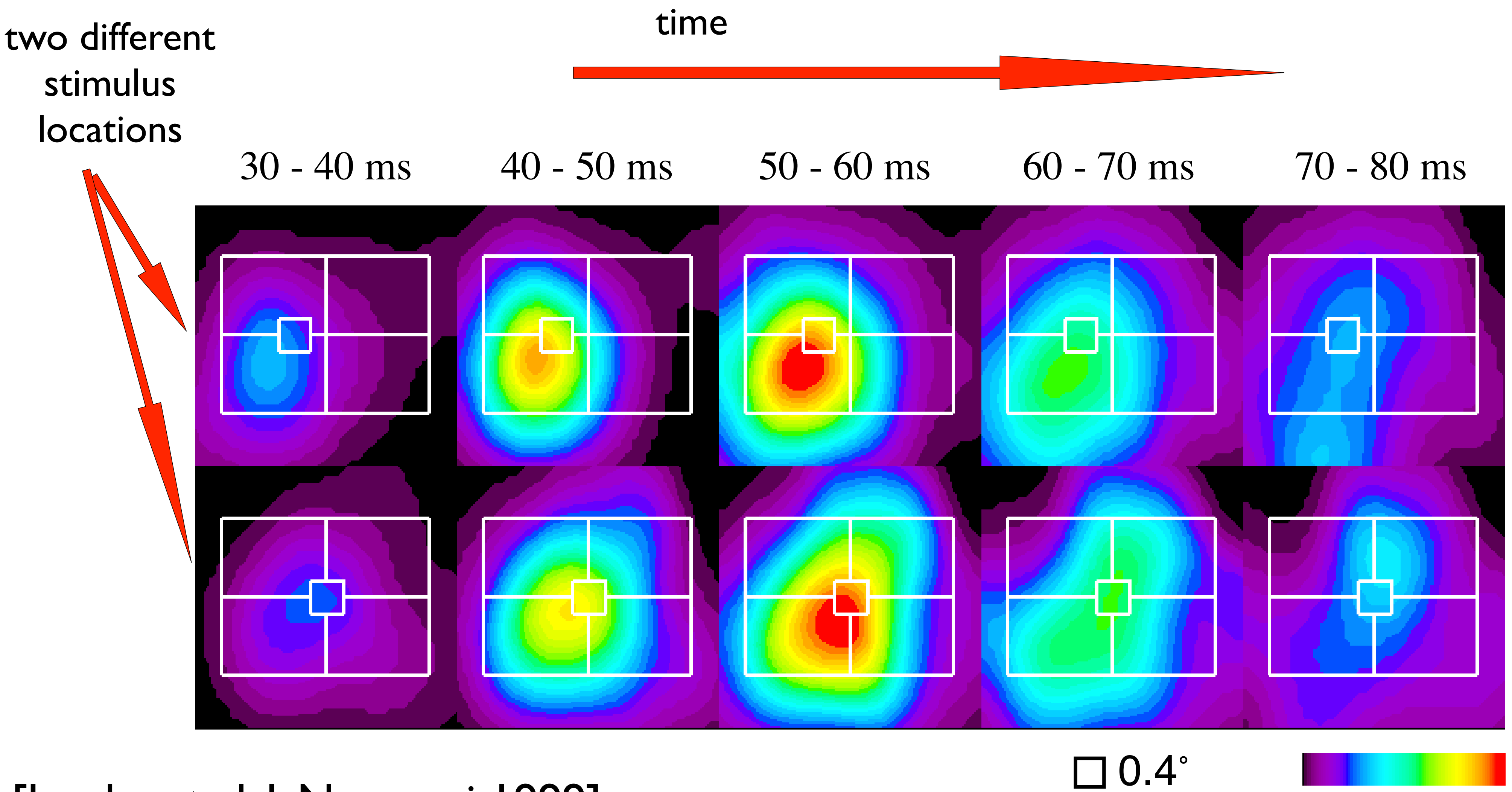
current stimulus:
square of light

range of retinal field
sampled by neurons



[Jancke et al, J. Neurosci 1999]

DPA: time course of activation

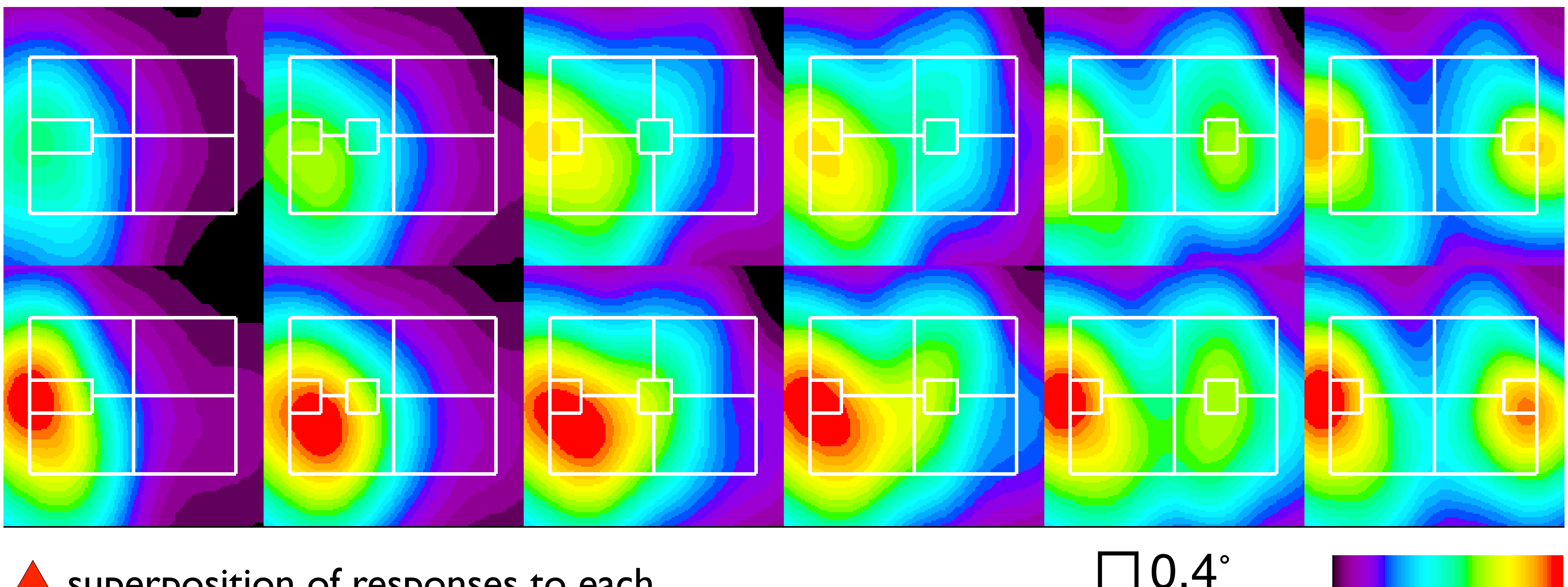


[Jancke et al, J. Neurosci 1999]

DPA: interaction

response to composite stimuli

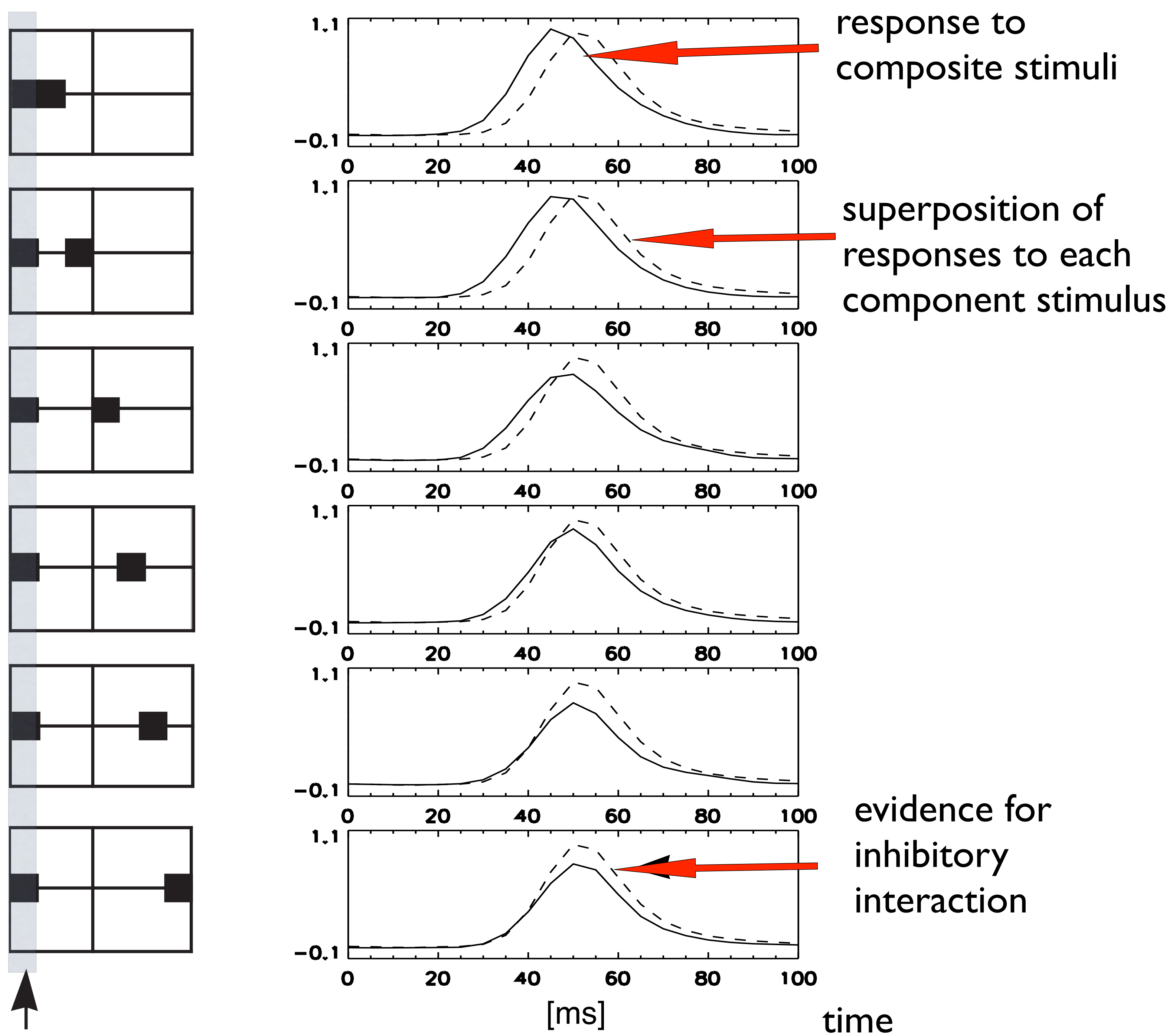
increasing distance between the two squares of light



superposition of responses to each component stimulus

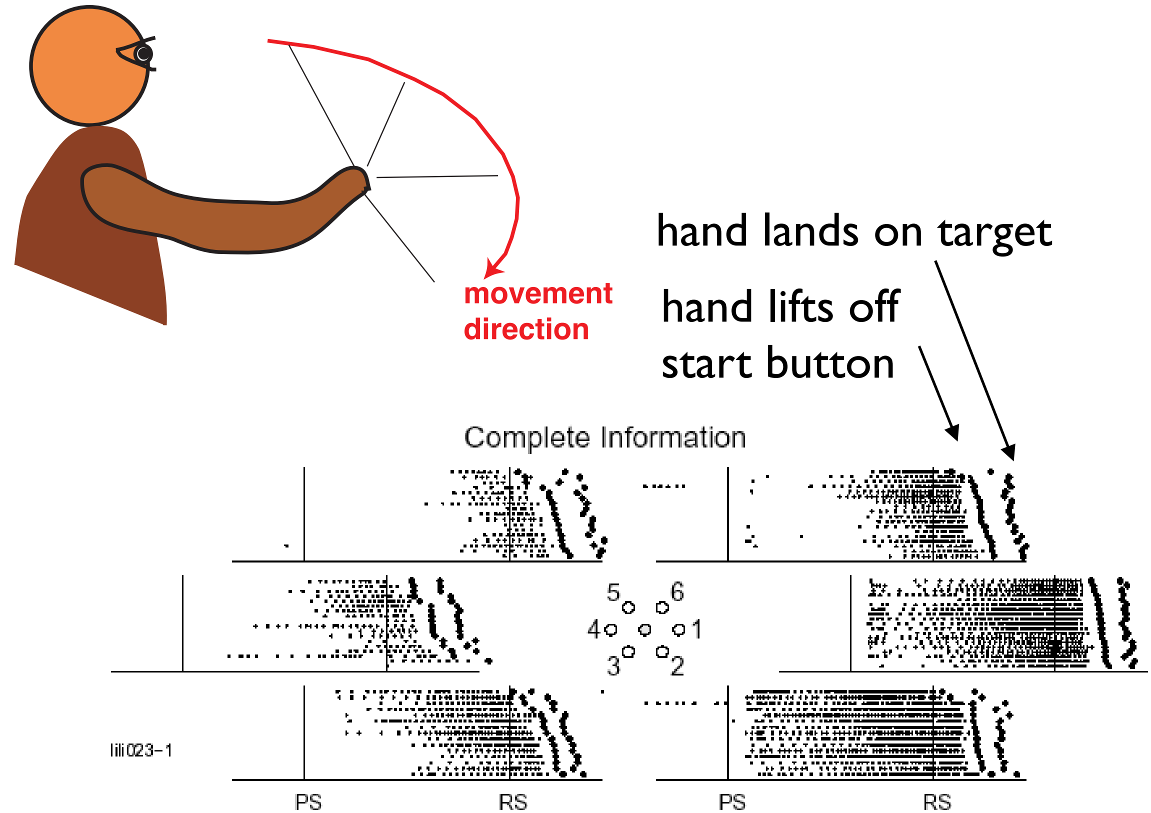
DPA: interaction

activation level in the DPA
at the location of the left component stimulus

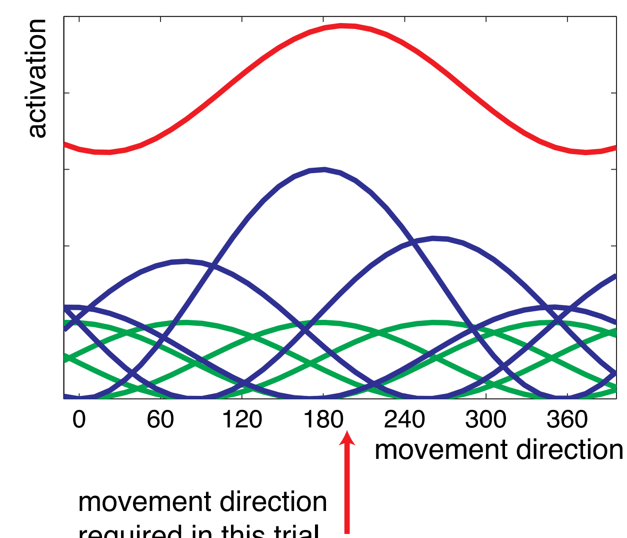


motor DPA

- motor: primary motor cortex
- determine tuning for each cell
- superpose these weighted by the current neural firing rate
- => DPA defined over movement direction



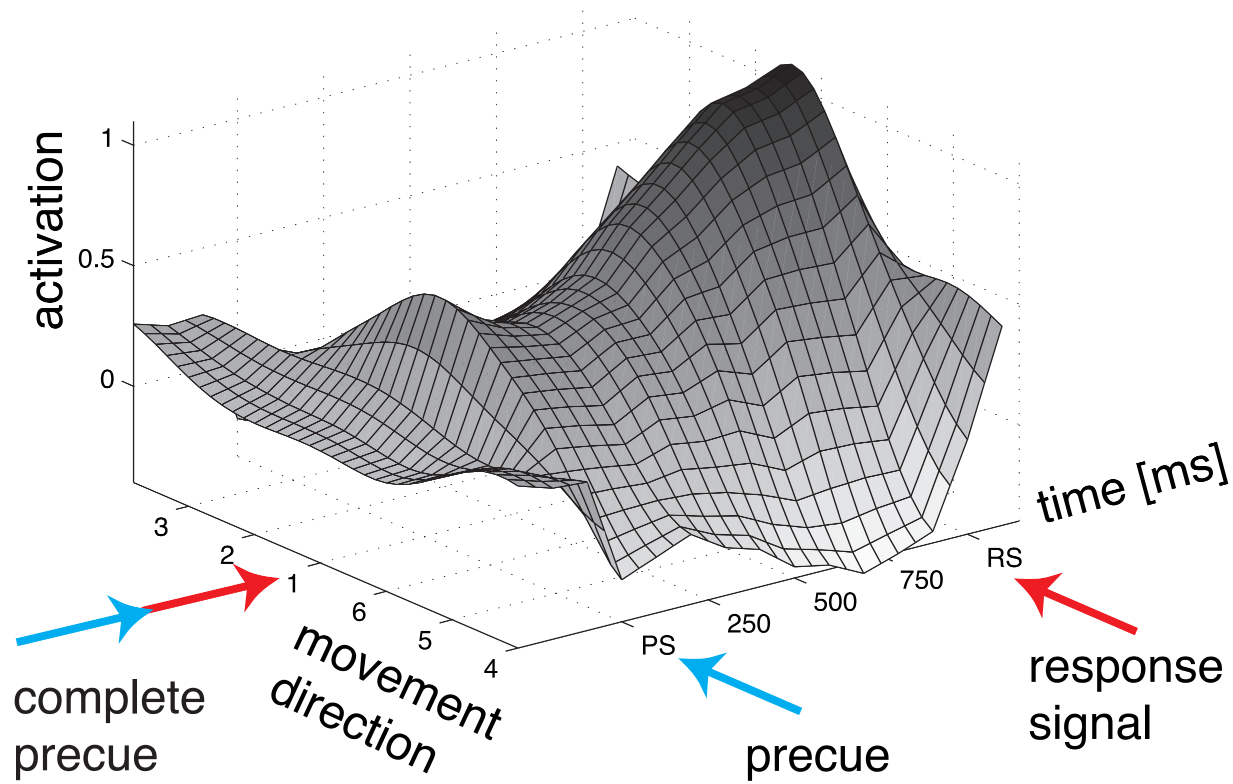
Distribution of population activation = $\sum_{\text{neurons}}$ tuning curve * current firing rate



[Bastian et al, Europ J. Neurosci 2003]

Motor DPA

- neurons contribute their entire tuning curve => distributed across field!

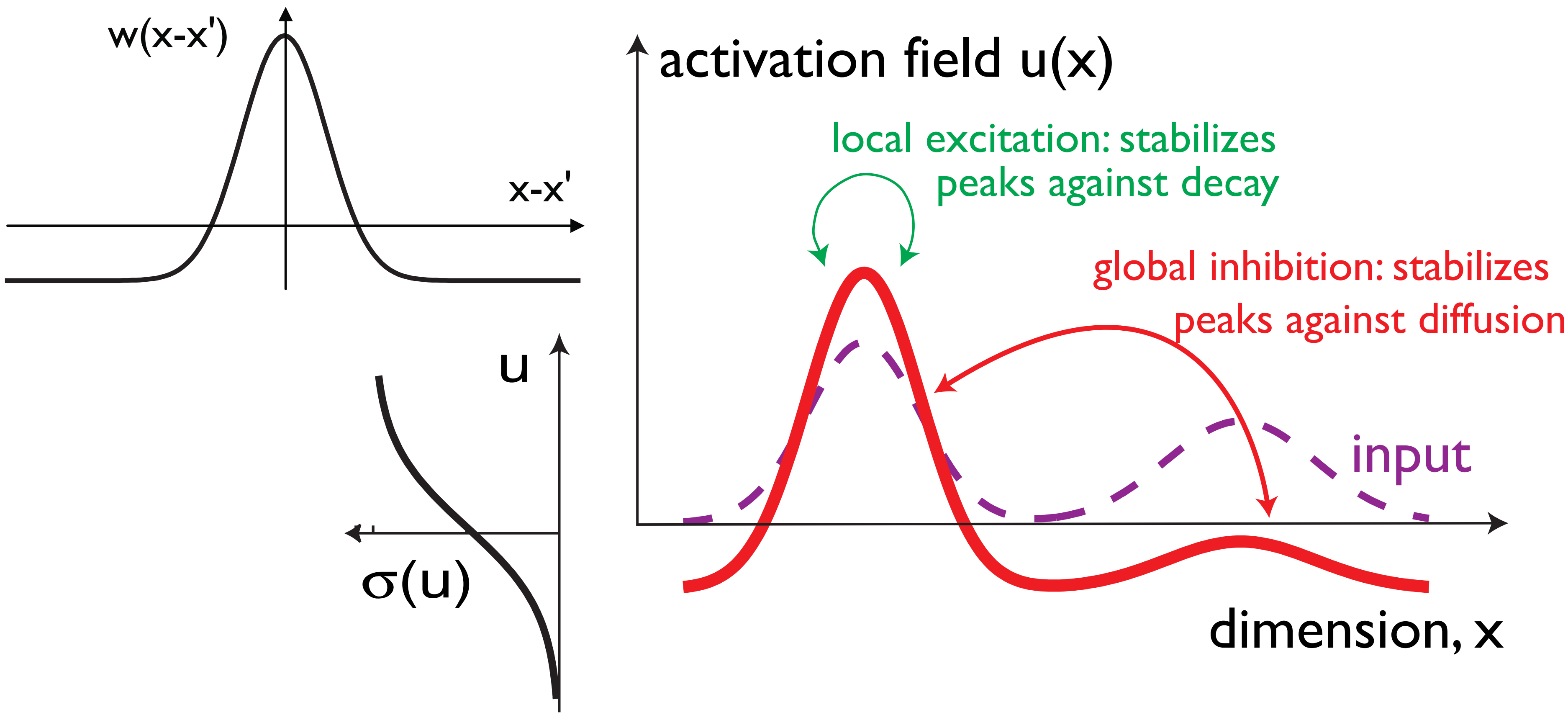


[Bastian et al, Europ J. Neurosci 2003]

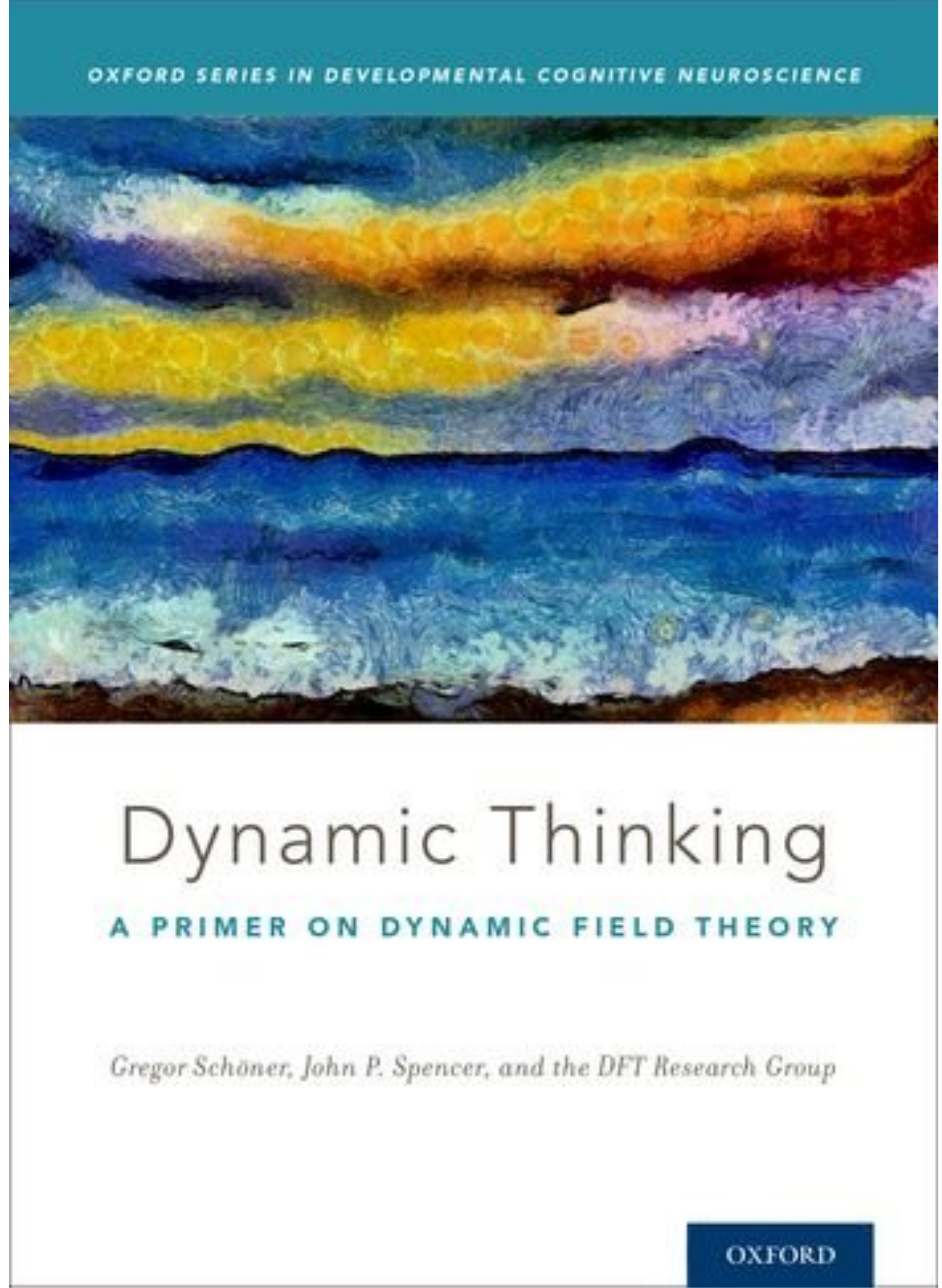
Neural dynamics

Neural dynamics of fields

- Peaks as stable states from intra-field interaction
- = local excitation/global inhibition



- dynamicfieldtheory.org



=> simulation

Attractors and their instabilities

- input driven solution (sub-threshold)
- self-stabilized solution (peak, supra-threshold)
- selection / selection instability
- working memory / memory instability
- boost-driven detection instability

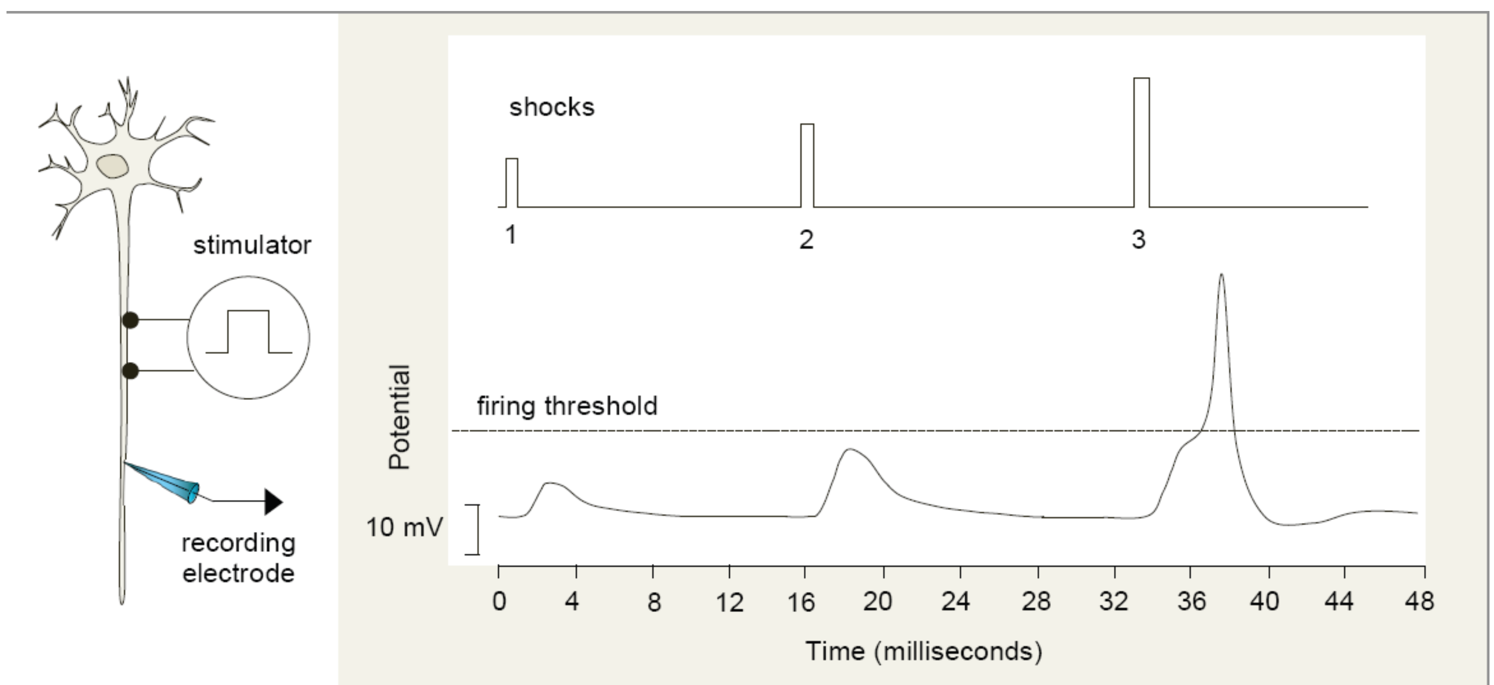
↓ detection instability

↑ reverse detection instability

Noise is critical near instabilities

The mathematics of neural dynamics

- dynamical state variable: activation, u, as a real number that reflects the (population) membrane potential

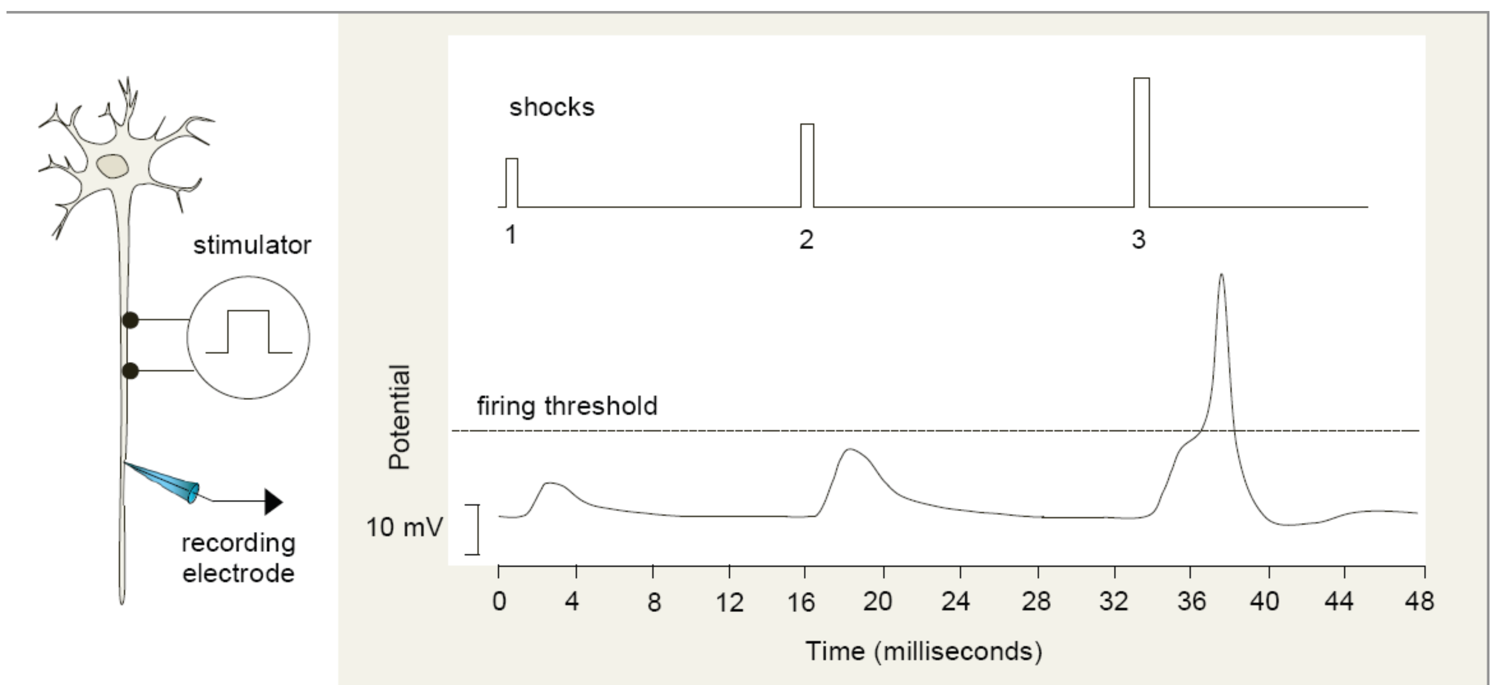


[from: Tresilian, 2012]

The mathematics of neural dynamics

- u(t) evolves as a dynamical system, characterized by a time scale, $\tau \approx 10\text{ms}$

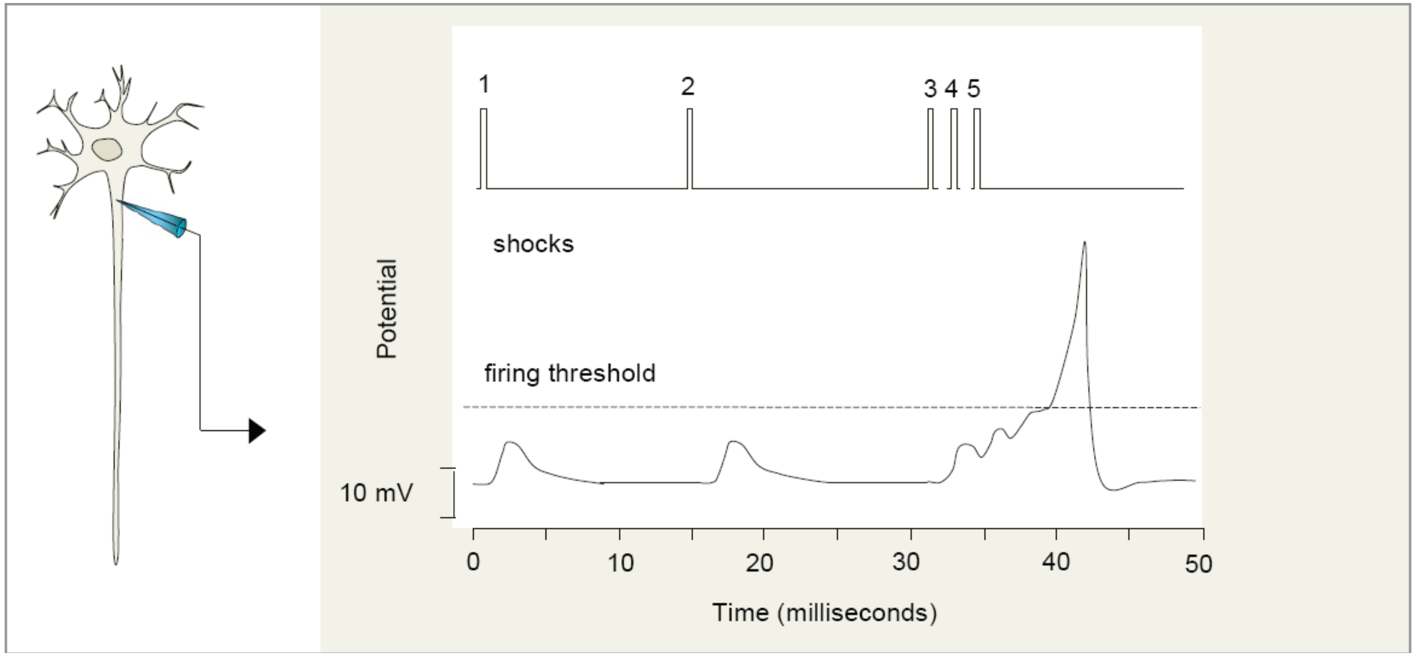
$$\tau \dot{u}(t) = -u(t) + h + \text{input}(t)$$



[from: Tresilian, 2012]

The mathematics of neural dynamics

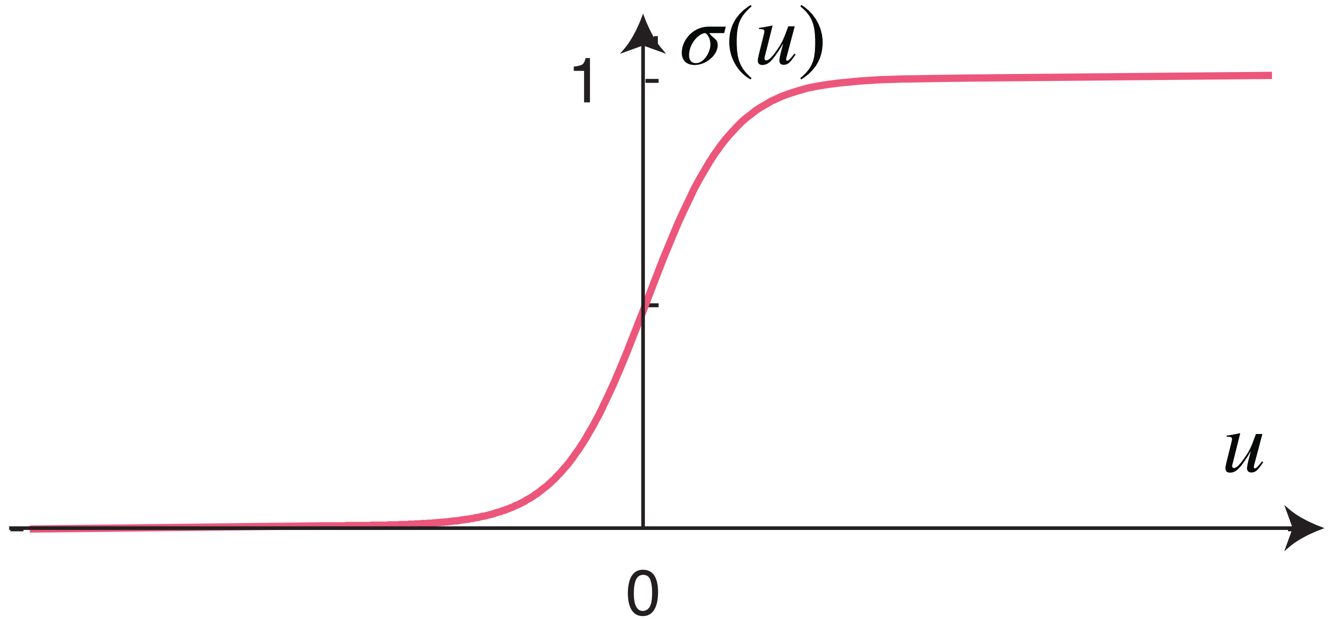
- spiking when membrane potential exceeds threshold….
- spike train is transmitted to downstream neurons



[from: Tresilian, 2012]

The mathematics of neural dynamics

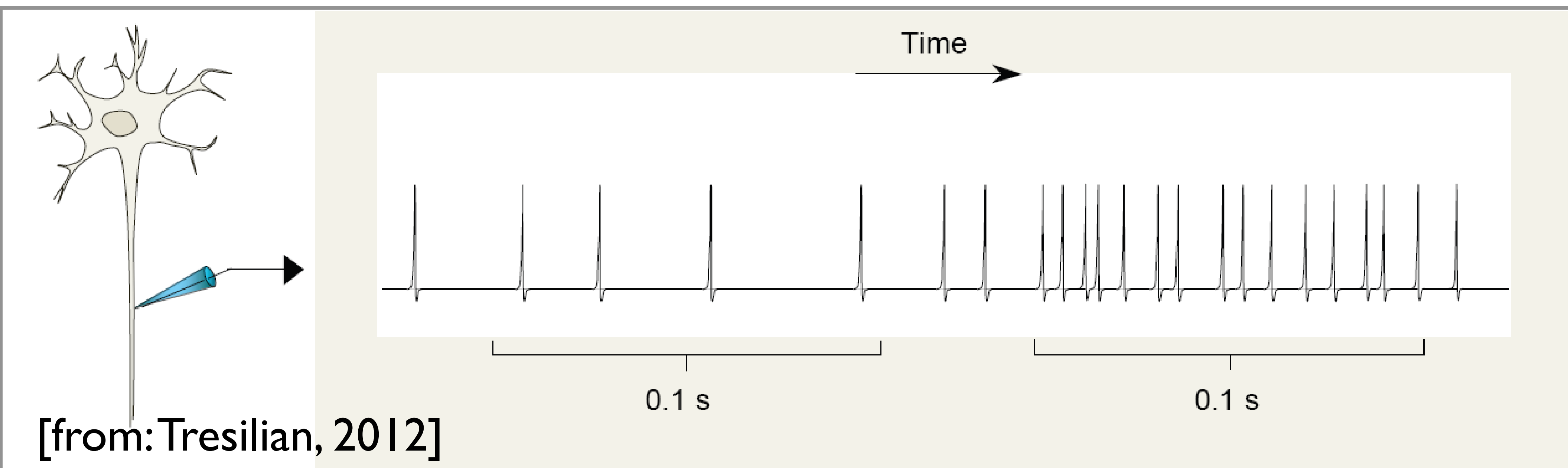
- in neural dynamics, that mechanism is replaced by a statistical (population) description: threshold function



[from: Tresilian, 2012]

The mathematics of neural dynamics

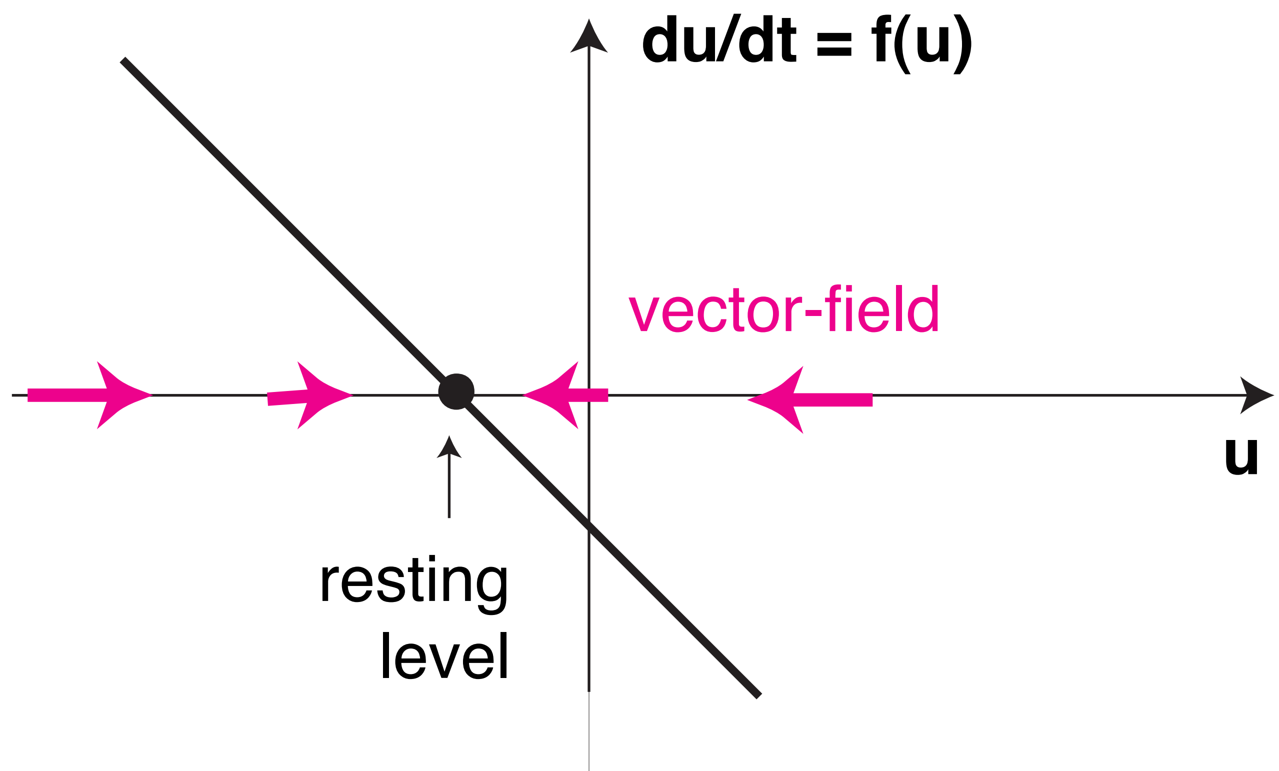
- that captures different firing rates in a small population…



[from: Tresilian, 2012]

Neural dynamics

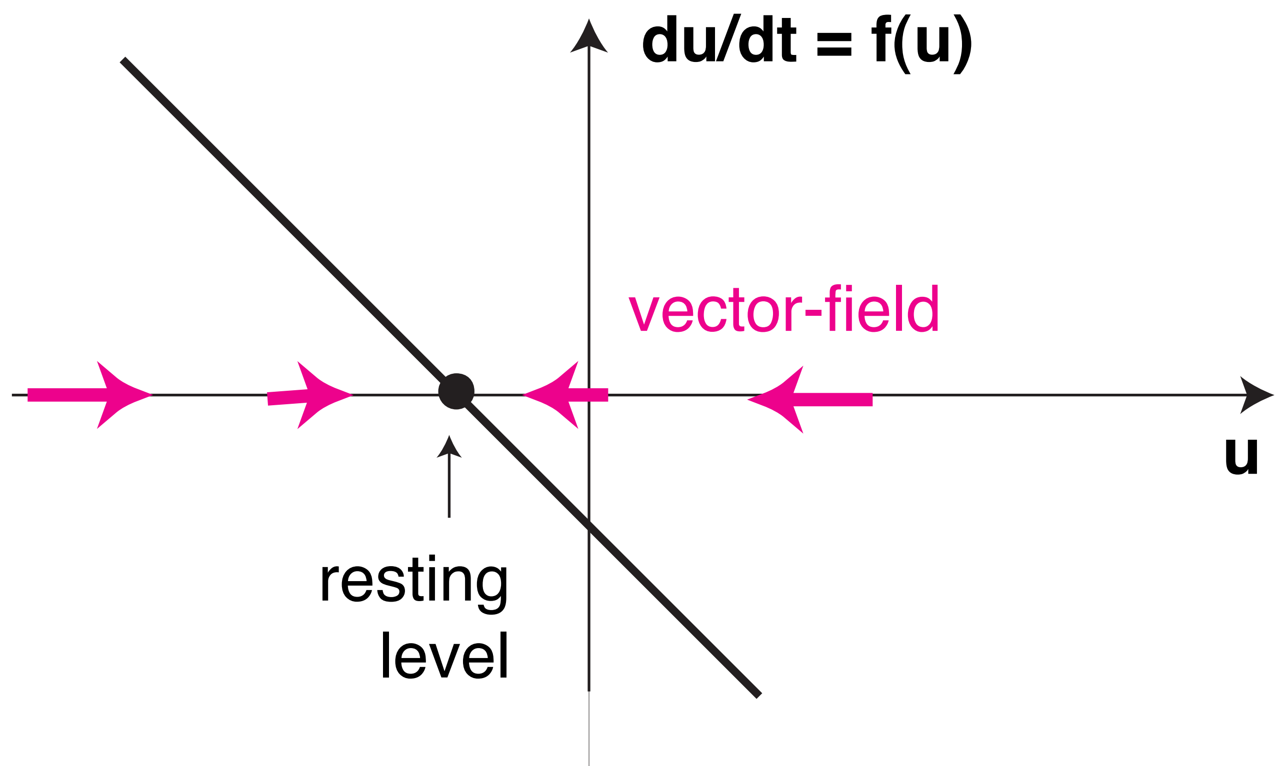
- dynamical system: the present predicts the future
- given a initial level of activation, u(0), the activation, u(t), at times t>0 is uniquely determined



$$\tau \dot{u}(t) = -u(t) + h$$

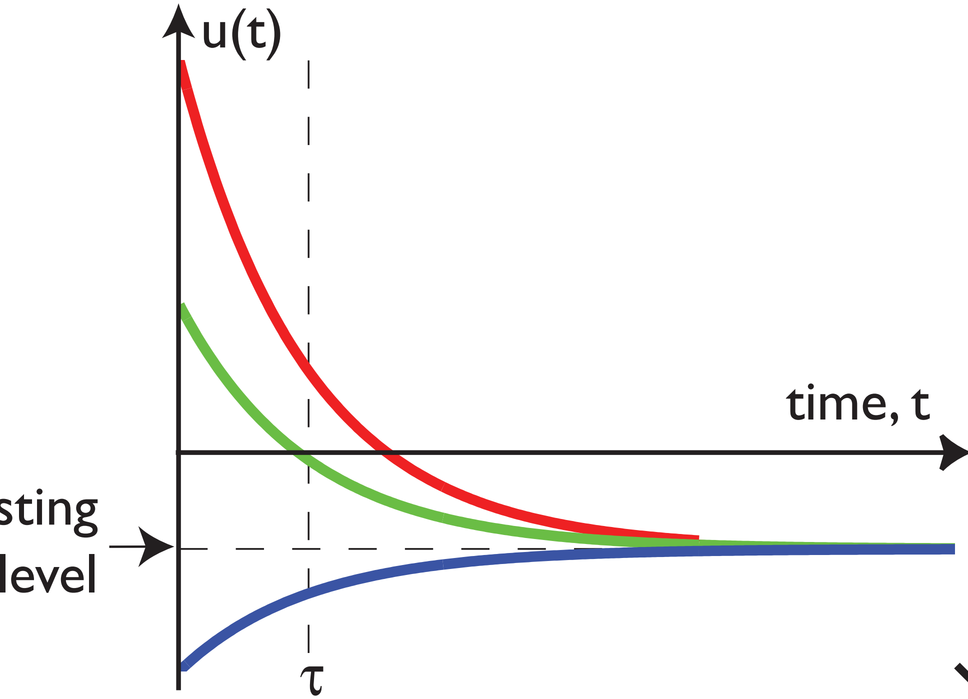
Neural dynamics

- fixed point = constant solution (stationary state)
- stable fixed point = attractor: nearby solutions converge to the fixed point



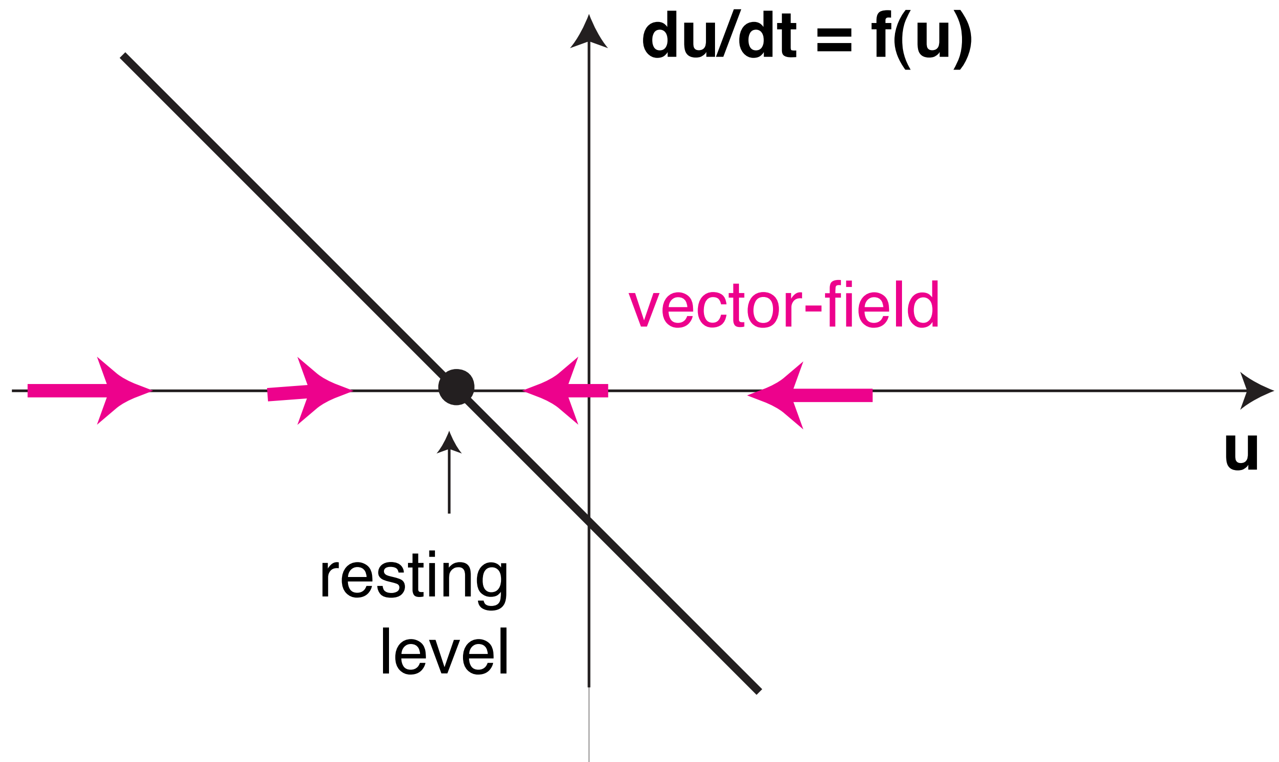
$$\tau\dot{u}(t) = -u(t) + h$$

Neural dynamics



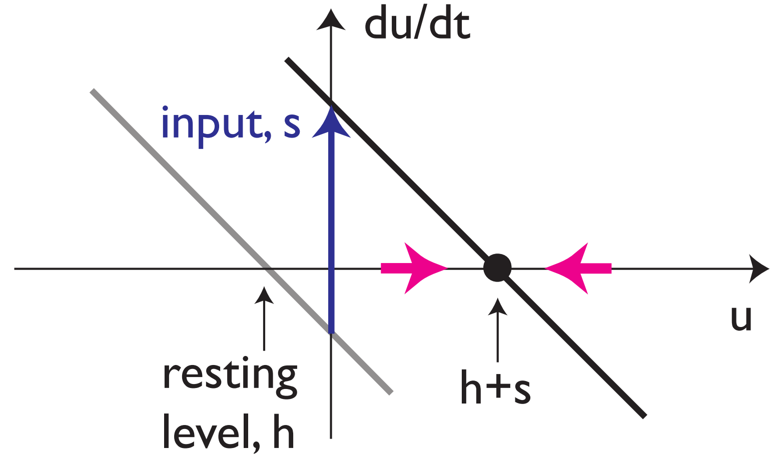
- attractors structure the ensemble of solutions (for all initial conditions) = flow

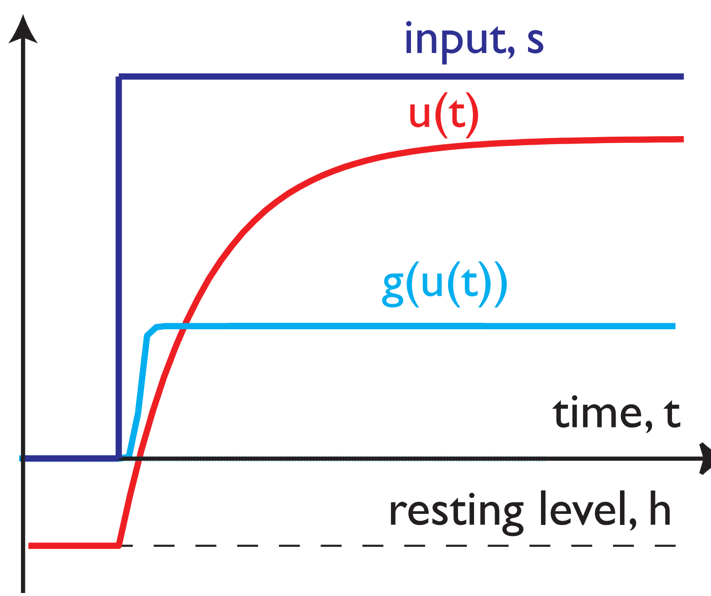
$$\tau \dot{u}(t) = -u(t) + h$$



Neuronal dynamics

- in neural dynamics, inputs are contributions to the rate of change
  - positive: excitatory
  - negative: inhibitory
- => shifts the attractor
- => activation tracks this shift due to stability

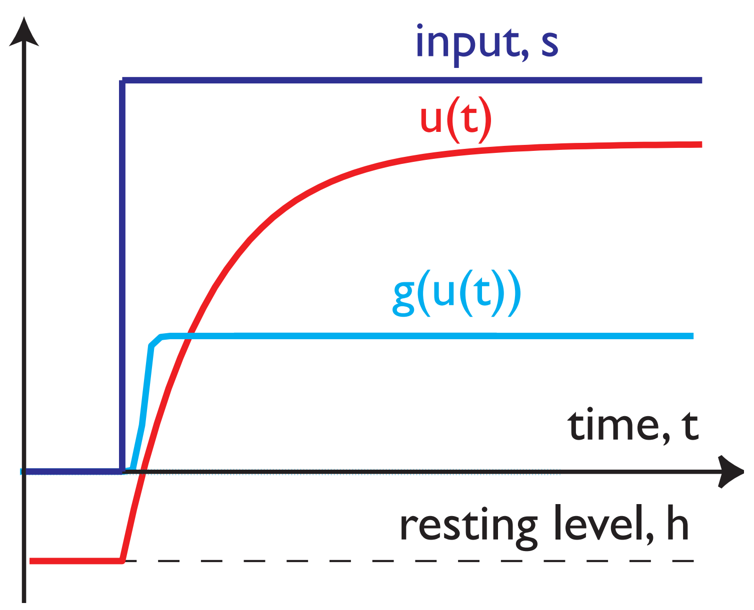




$$\tau \dot{u}(t) = -u(t) + h + s(t)$$

Neuronal dynamics

- what is transmitted is $\sigma(u(t))$
- (labelled g(t) in the book and in some figures)
- => neural dynamics as a low-pass filter of time varying input
- = input-driven solution

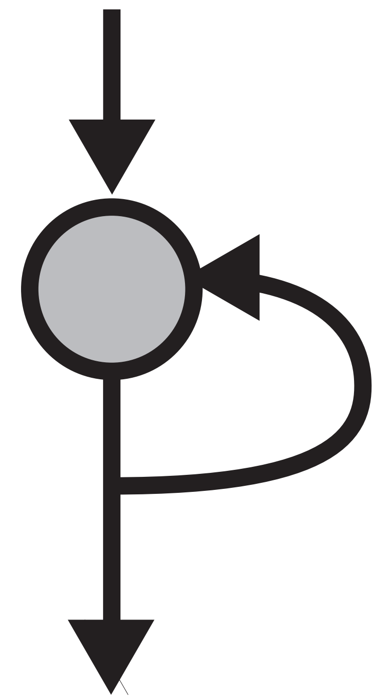


$$\tau \dot{u}(t) = -u(t) + h + s(t)$$

=> simulation

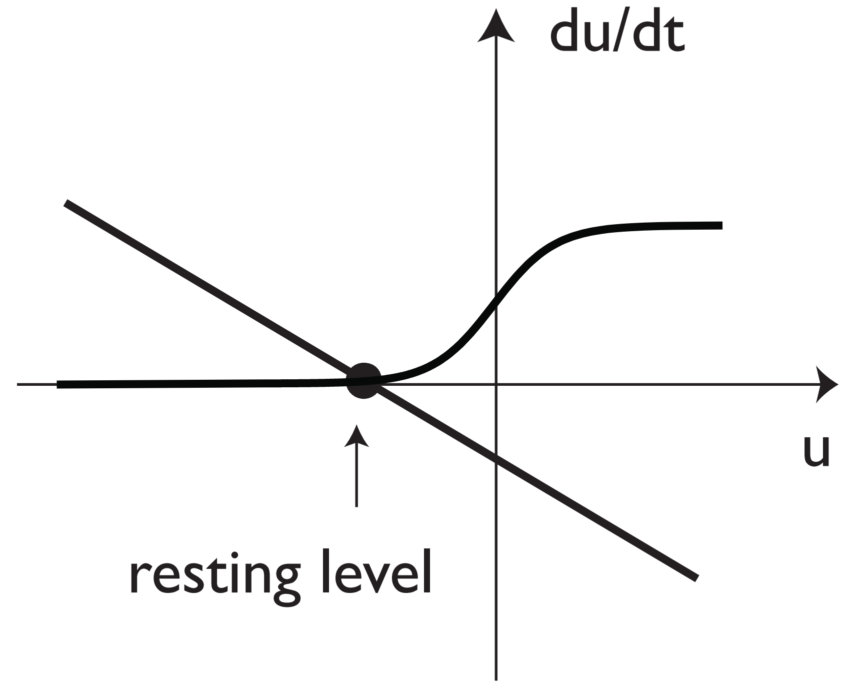
Neuronal dynamics with self-excitation

- single activation variable with self-excitation
- representing a small population with excitatory coupling

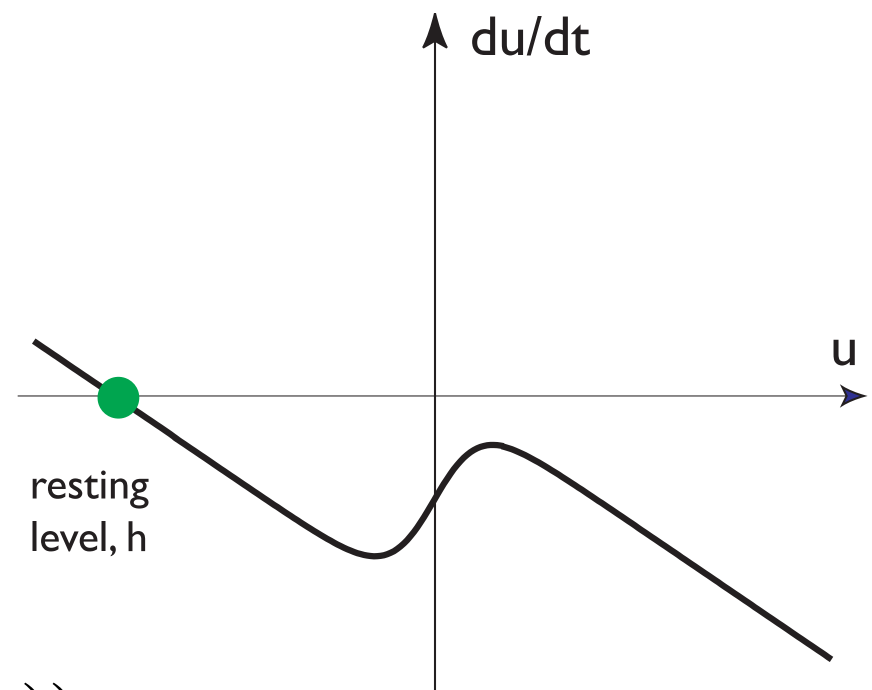


$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation



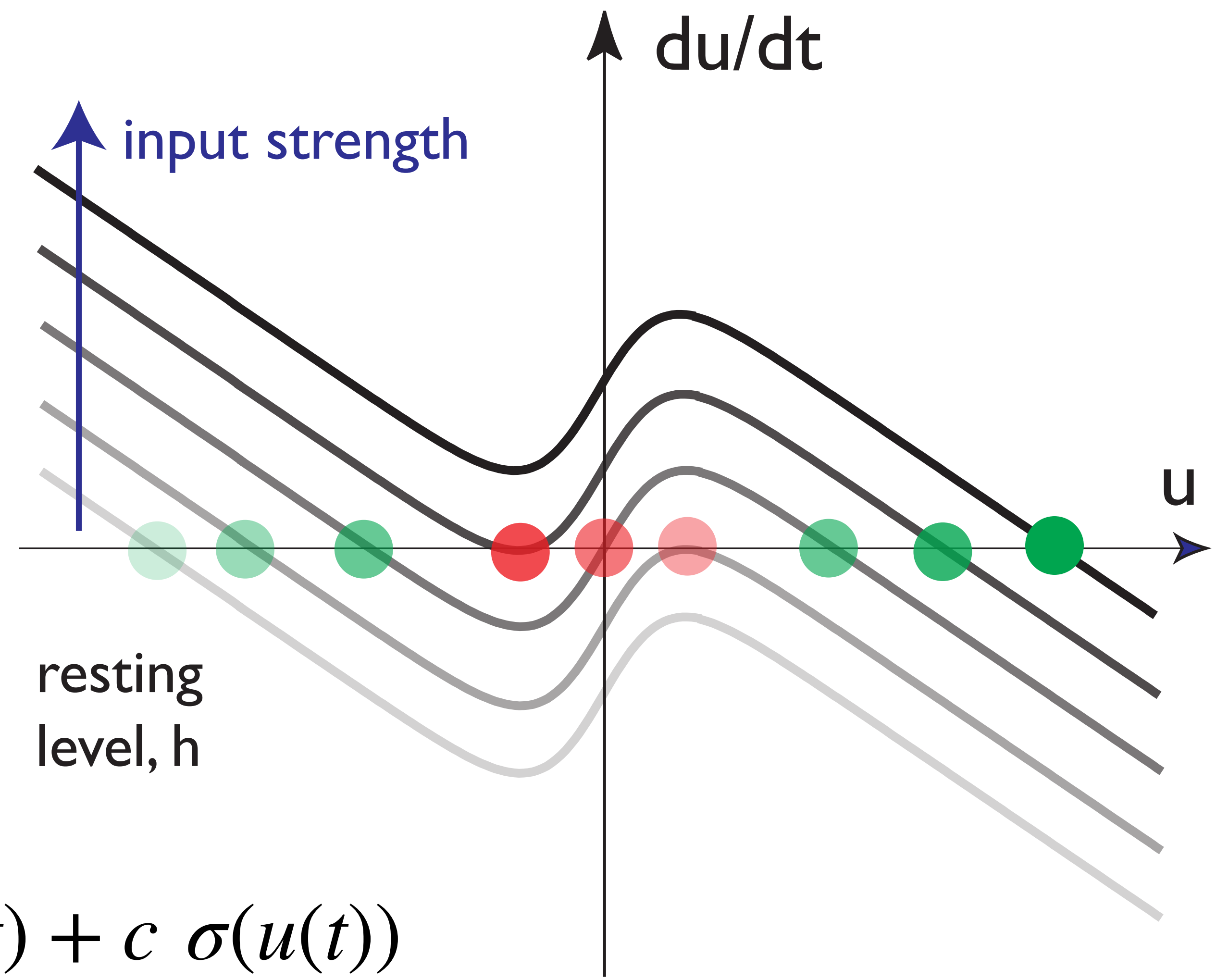
- => nonlinear dynamics!



$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

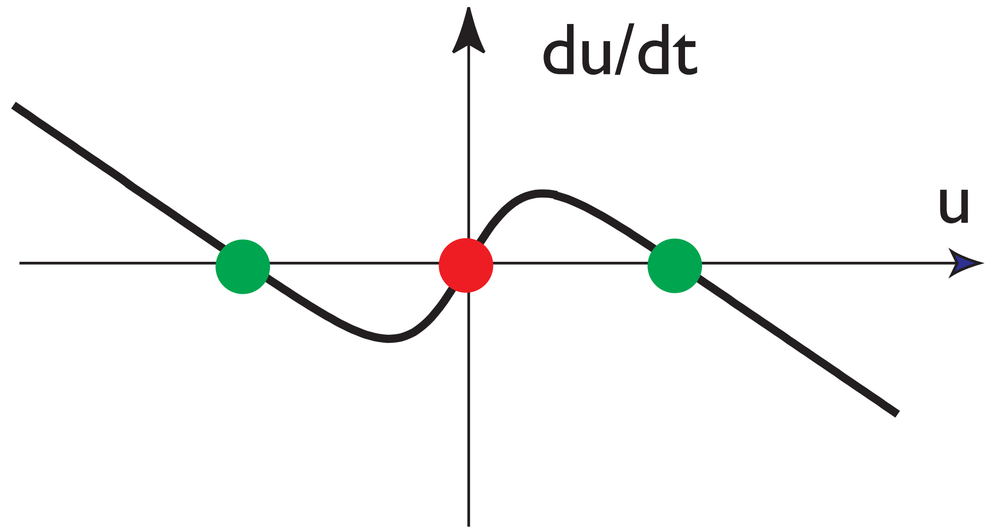
Neuronal dynamics with self-excitation

- varying input

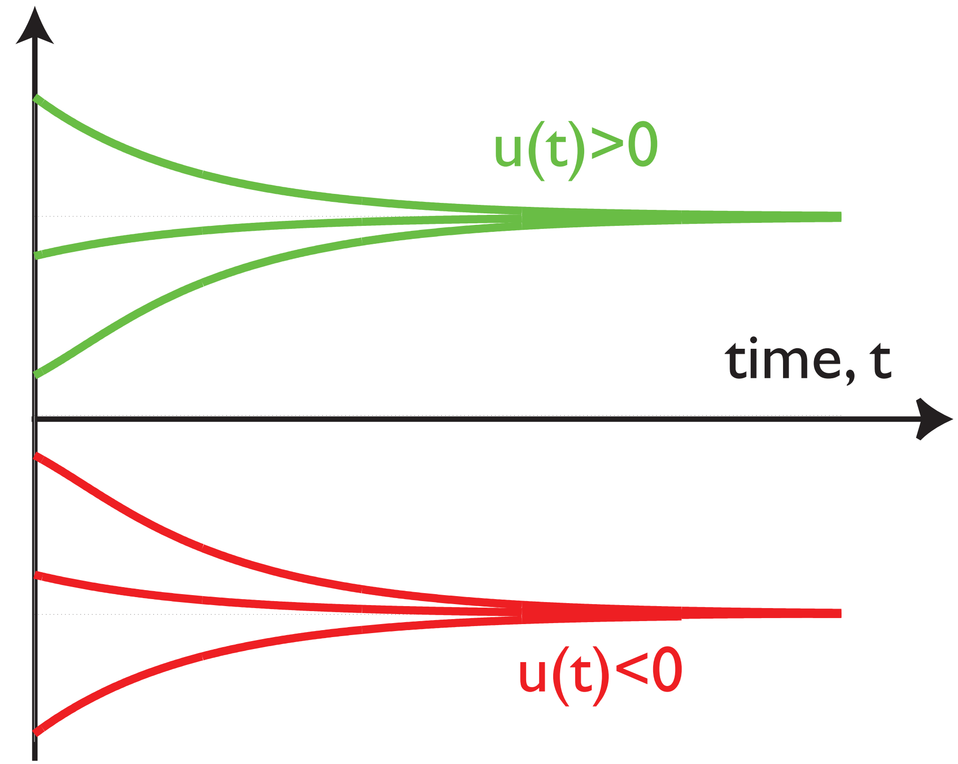


$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation



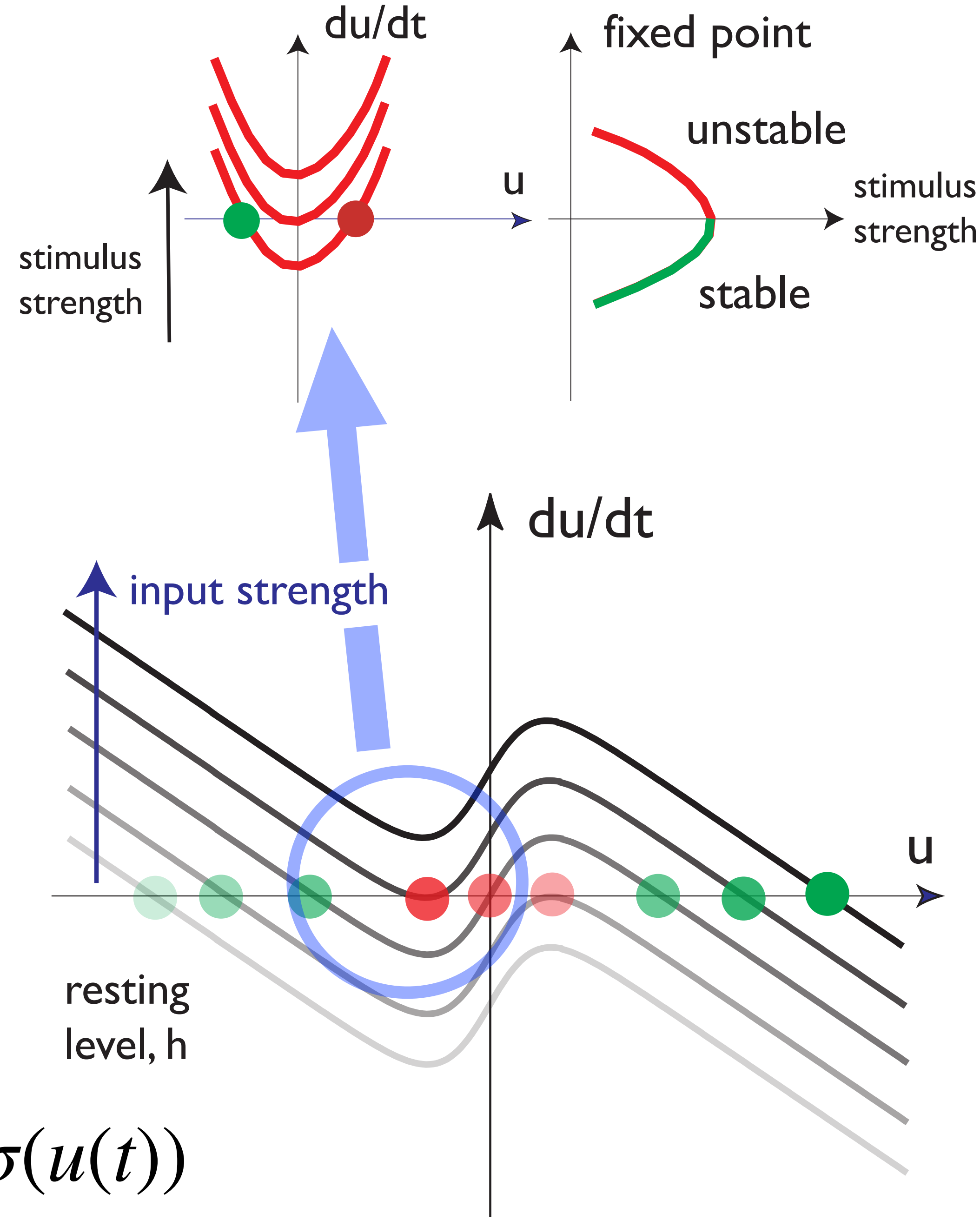
- at intermediate stimulus strength: bistable
- “on” vs “off” state



$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

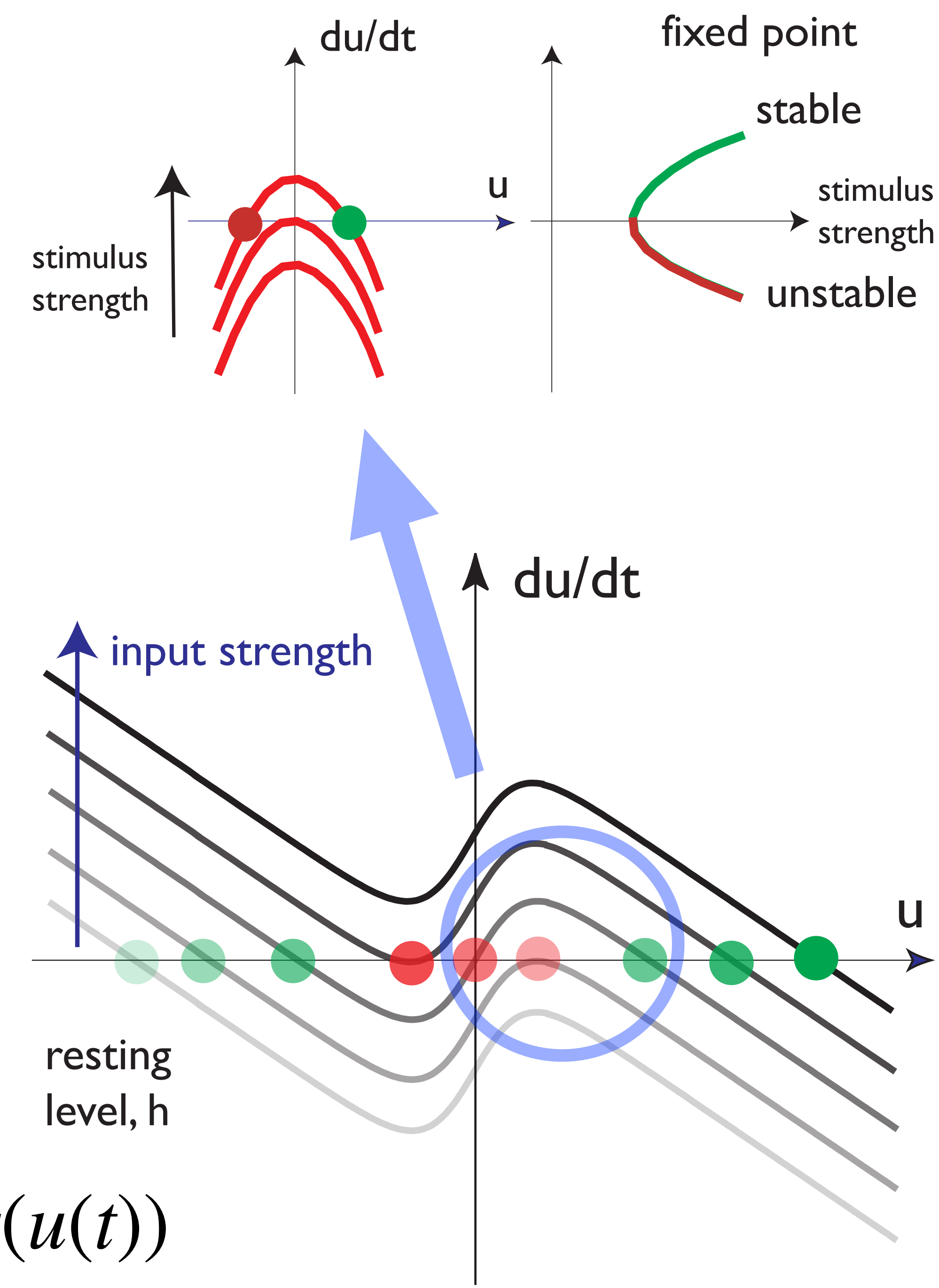
Neuronal dynamics with self-excitation

- increasing input strength => detection instability
- => the detection decision is stabilized



$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation

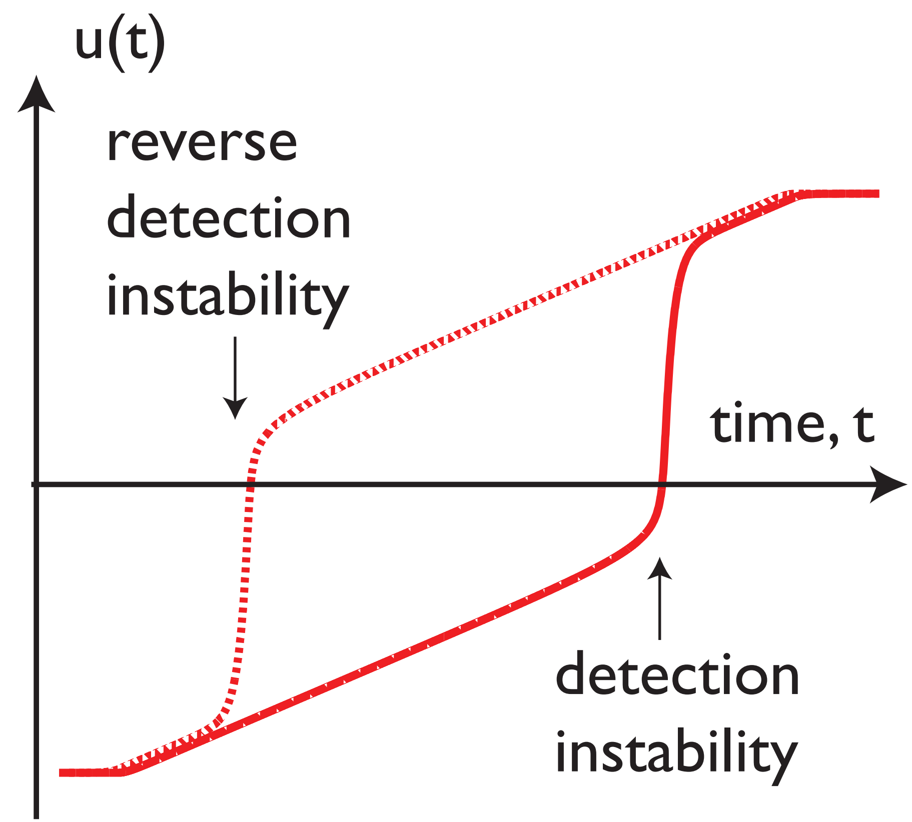


- decreasing input strength => reverse detection instability

$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation

- the detection and its reverse => create discrete events from time-continuous changes

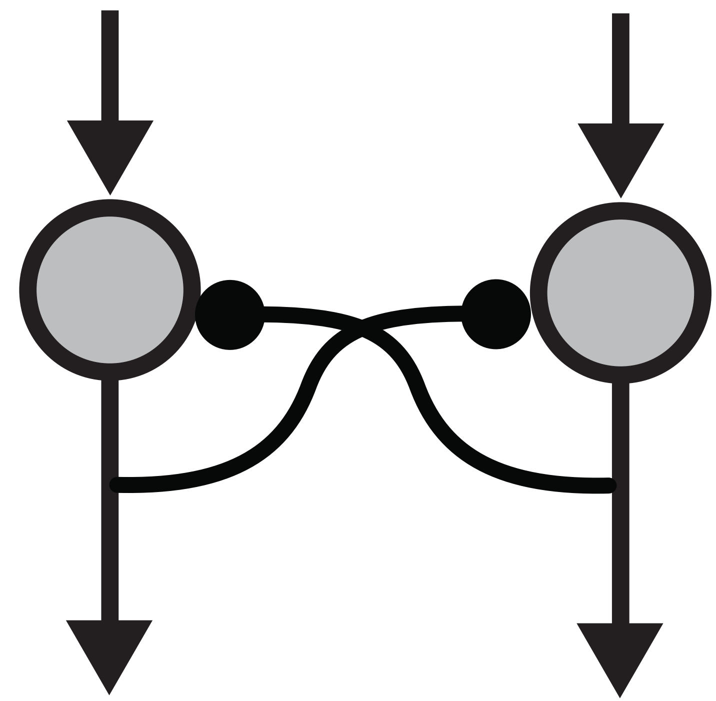


$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

=> simulation

Neuronal dynamics with competition

- two activation variables with reciprocal inhibitory coupling
- representing two small populations that are inhibitorily coupled

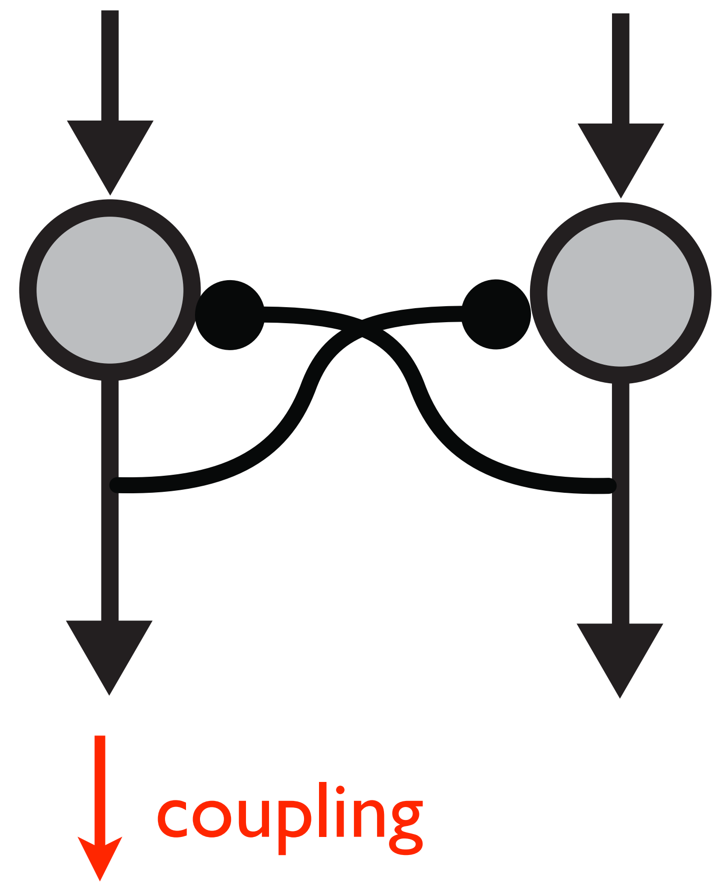


$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - \sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - \sigma(u_1(t))$$

Neuronal dynamics with competition

- Coupling: the rate of change of one activation variable depends on the level of activation of the other activation variable

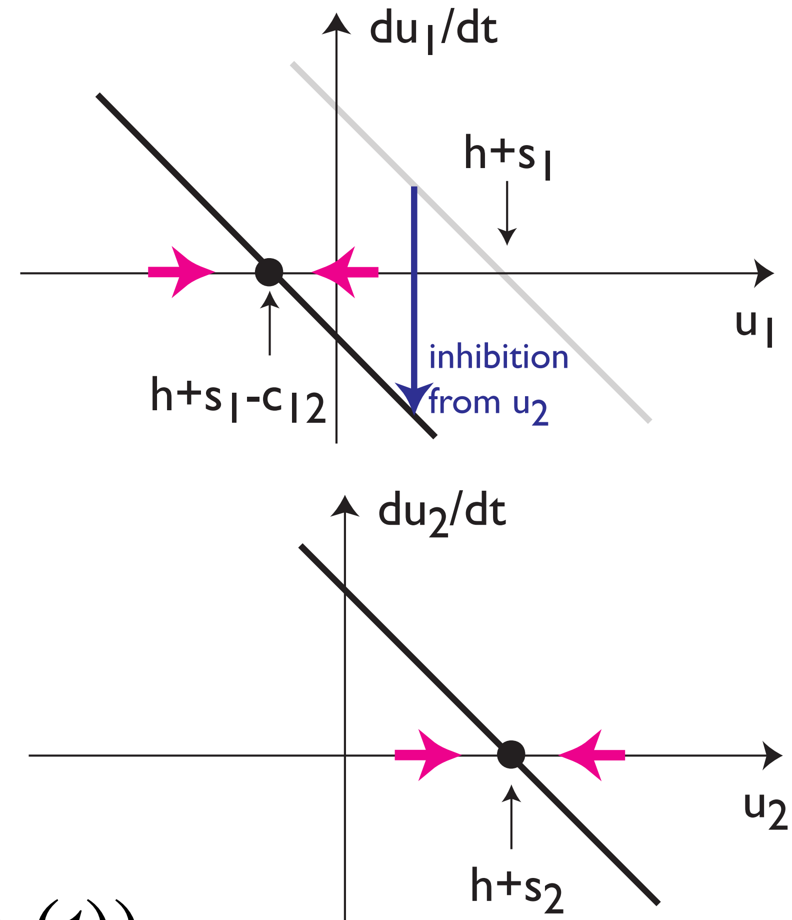


$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - \sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - \sigma(u_1(t))$$

Neuronal dynamics with competition

- to visualize, assume that u_2 has been activated by input to a positive level
- => it inhibits u_1

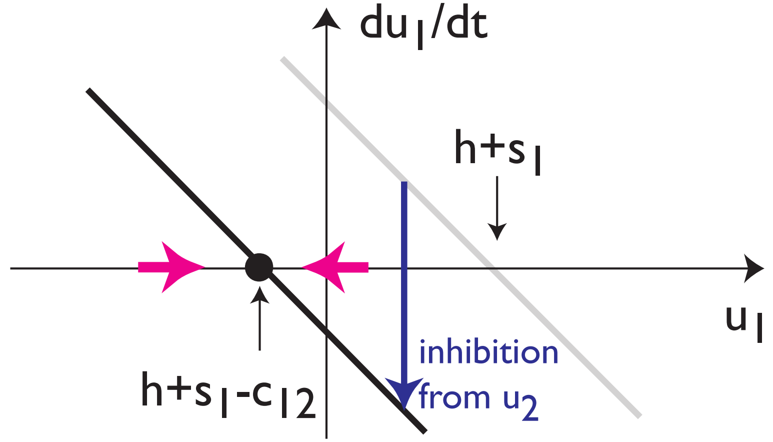


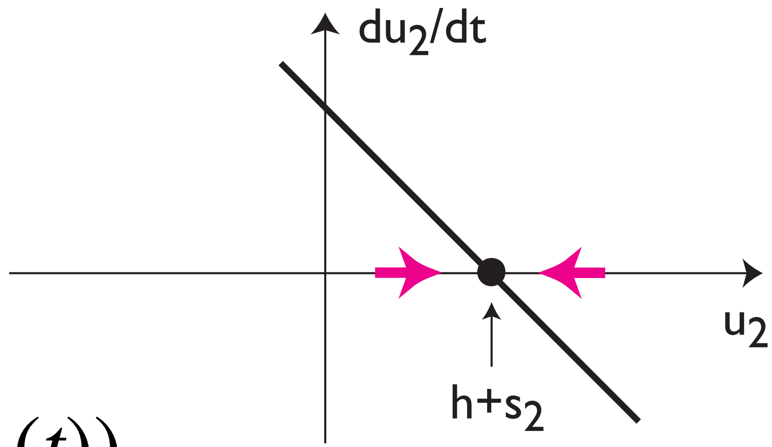
$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - \sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - \sigma(u_1(t))$$

Neuronal dynamics with competition

- why would u_2 be positive before u_1?
- more input to u_2 (better "match") => faster increase
- input advantage <=> time advantage <=> competitive advantage



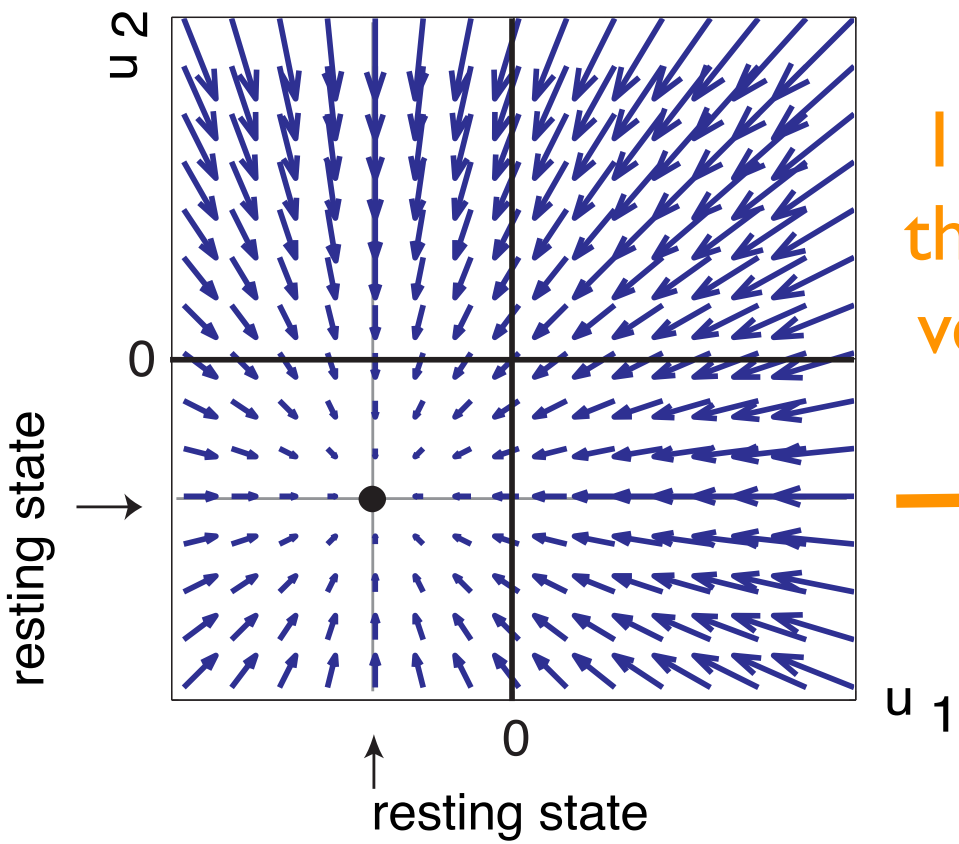


$$\tau\dot{u}_1(t) = -u_1(t) + h + s_1(t) - \sigma(u_2(t))$$

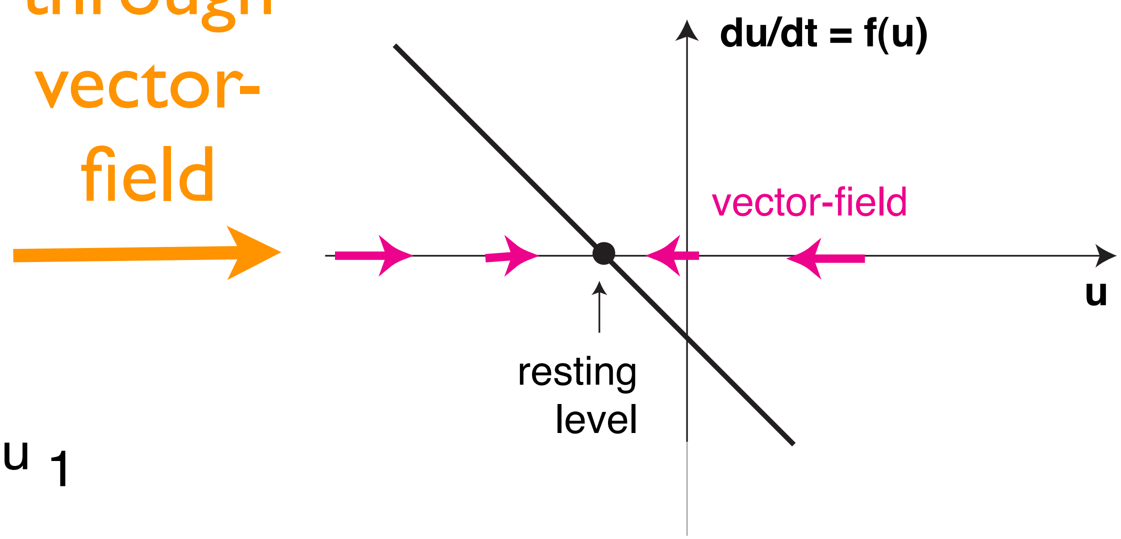
$$\tau\dot{u}_2(t) = -u_2(t) + h + s_2(t) - \sigma(u_1(t))$$

Neuronal dynamics with competition

vector-field in the absence of input

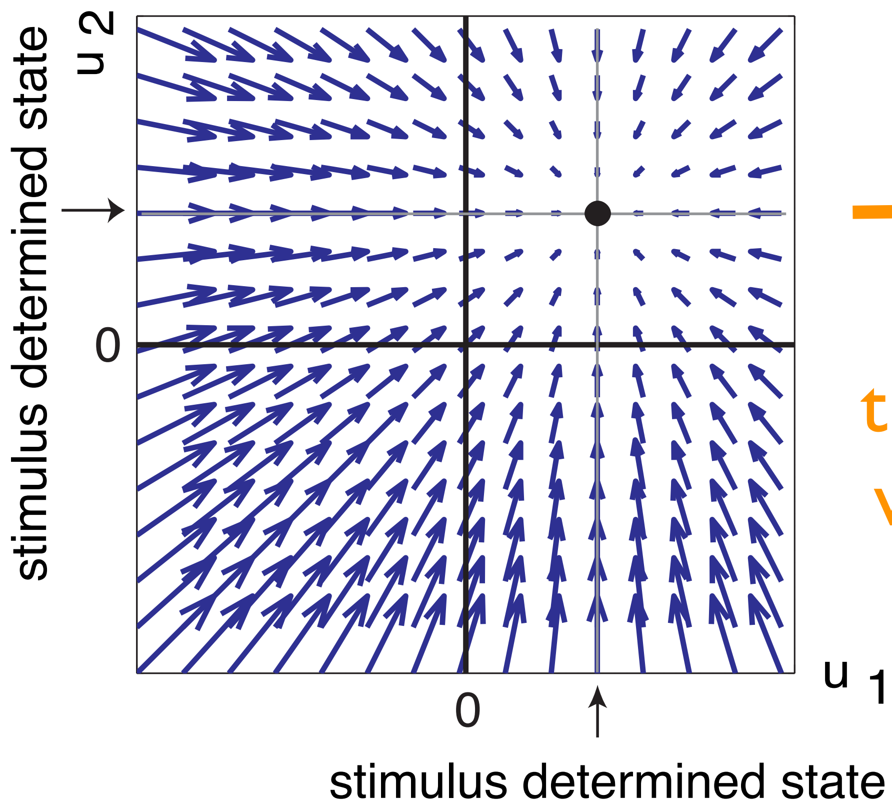


1D cut through vector-field

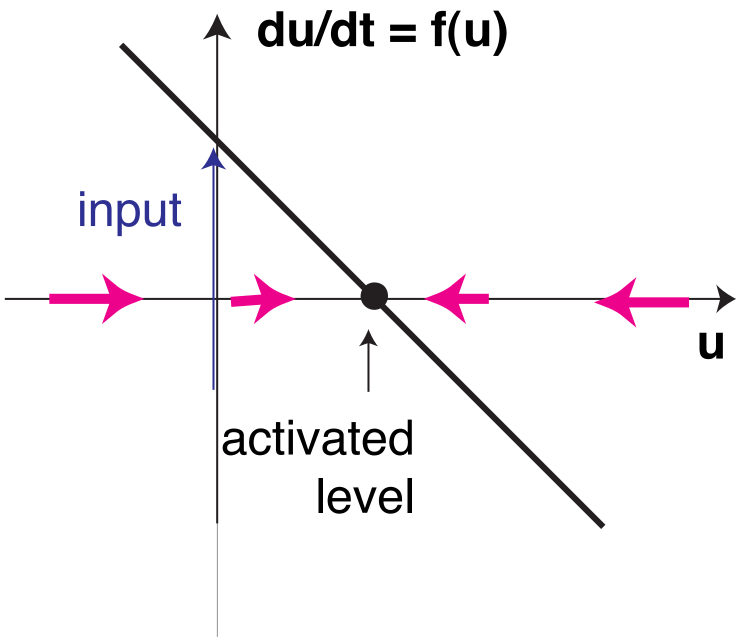


Neuronal dynamics with competition

vector-field (without interaction) when both neurons receive input

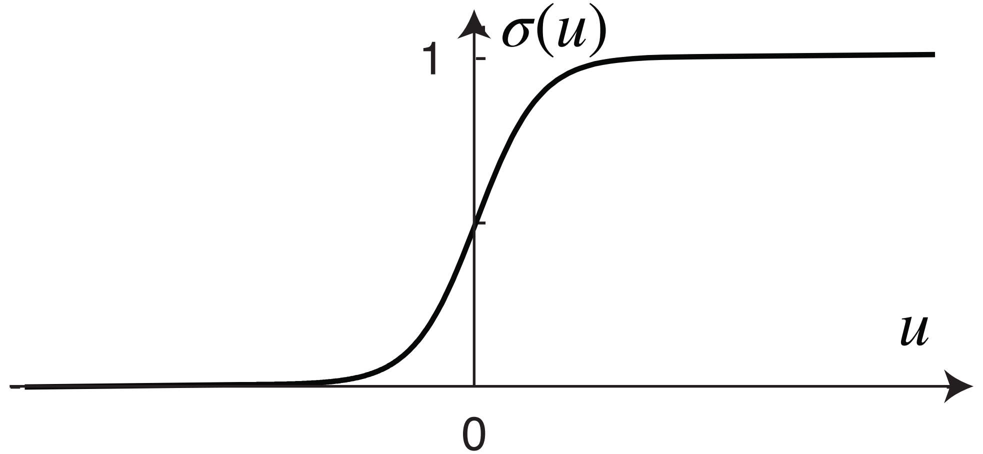


1D cut through vector-field



Neuronal dynamics with competition

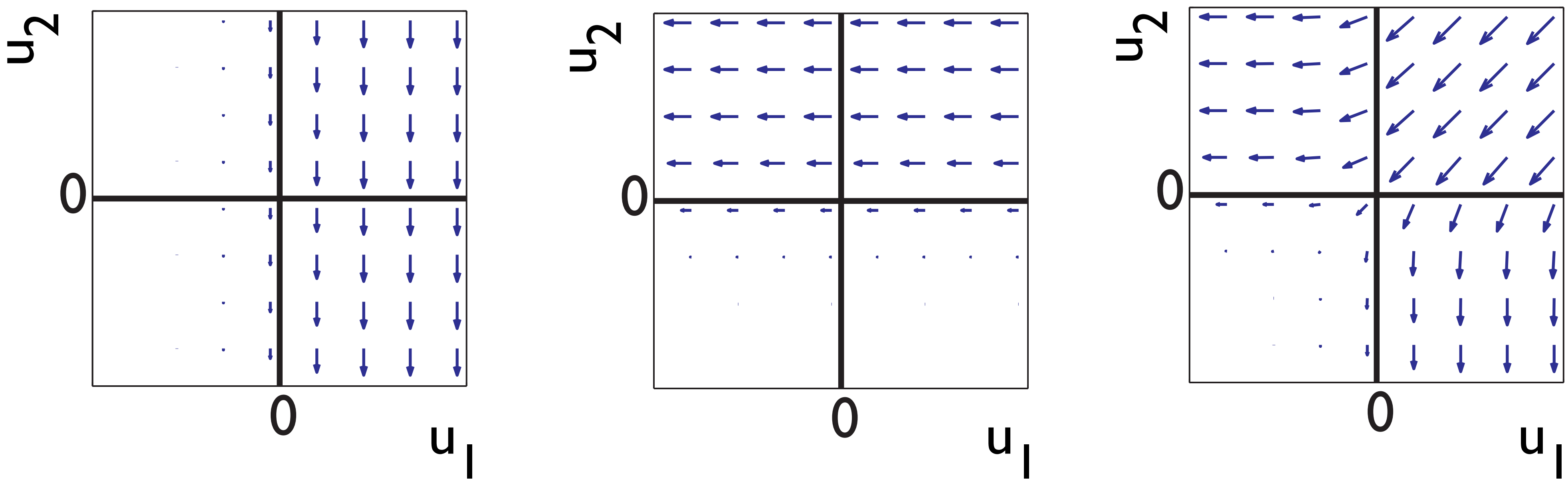
- only activated neurons participate in interaction!



Neuronal dynamics with competition

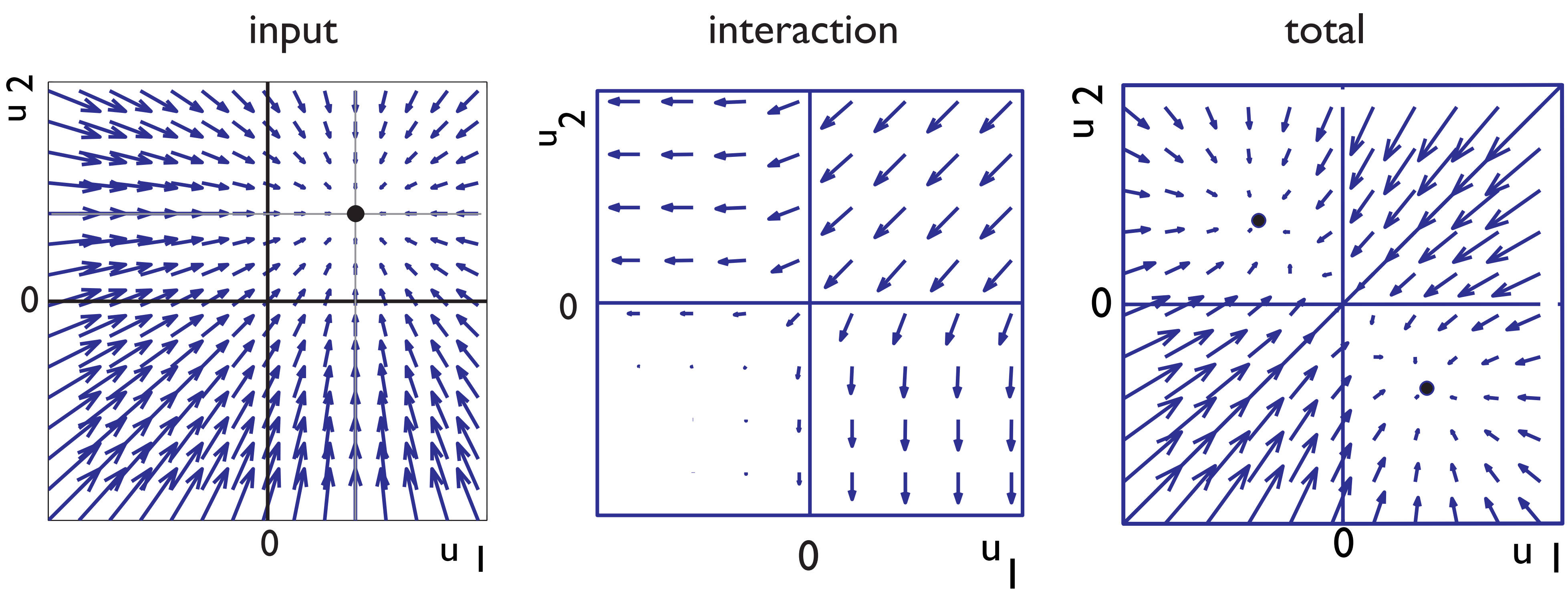
- vector-field of mutual inhibition

site 1 inhibits site 2 | site 2 inhibits site 1 | interaction combined



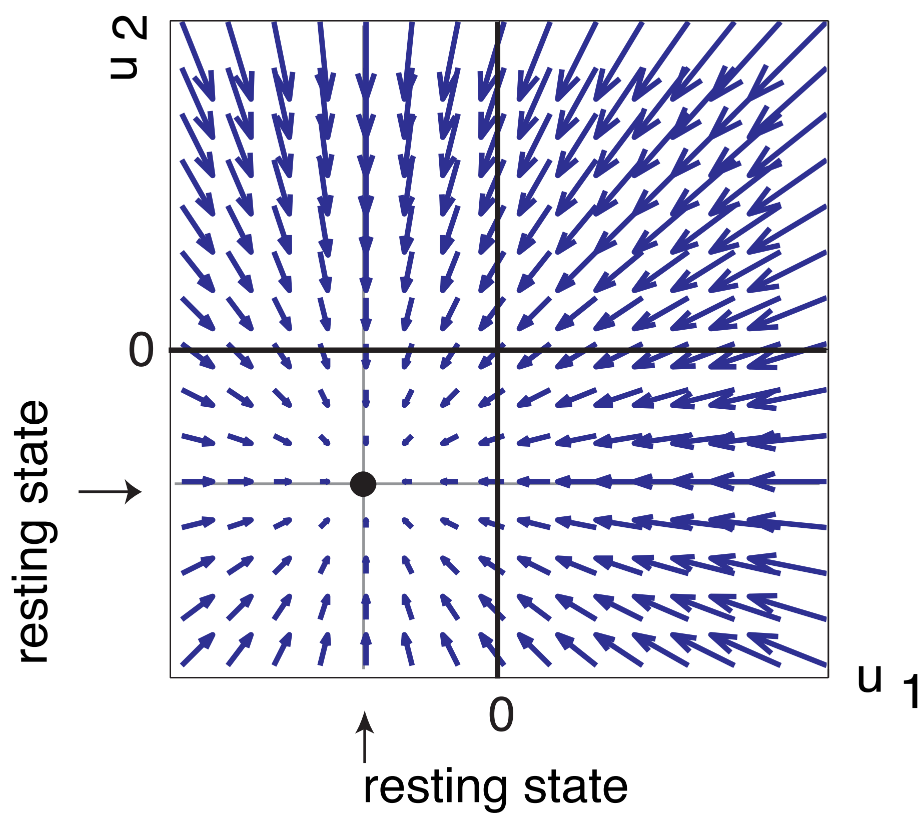
Neuronal dynamics with competition

vector-field with strong mutual inhibition: bistable

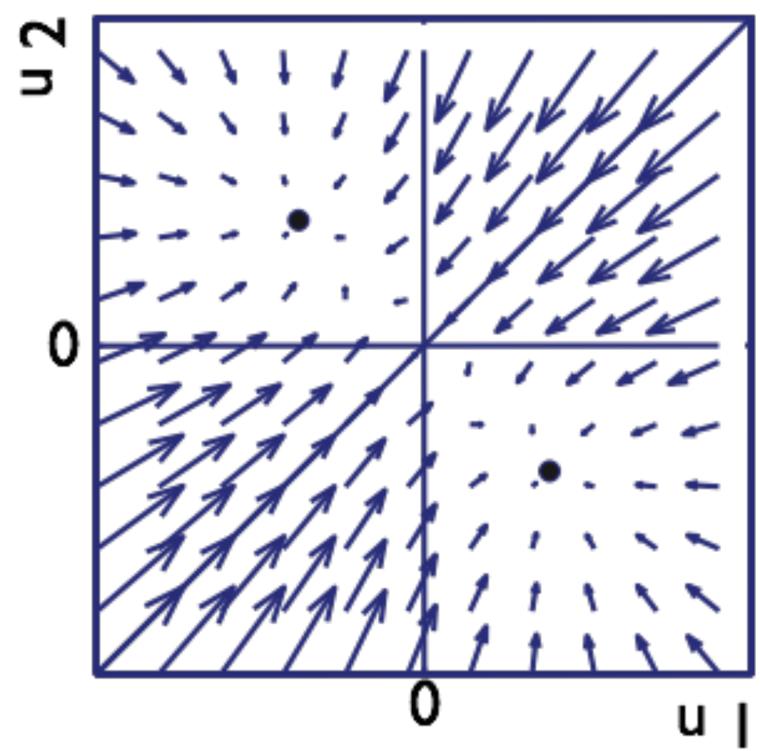


Neuronal dynamics with competition

before input is presented

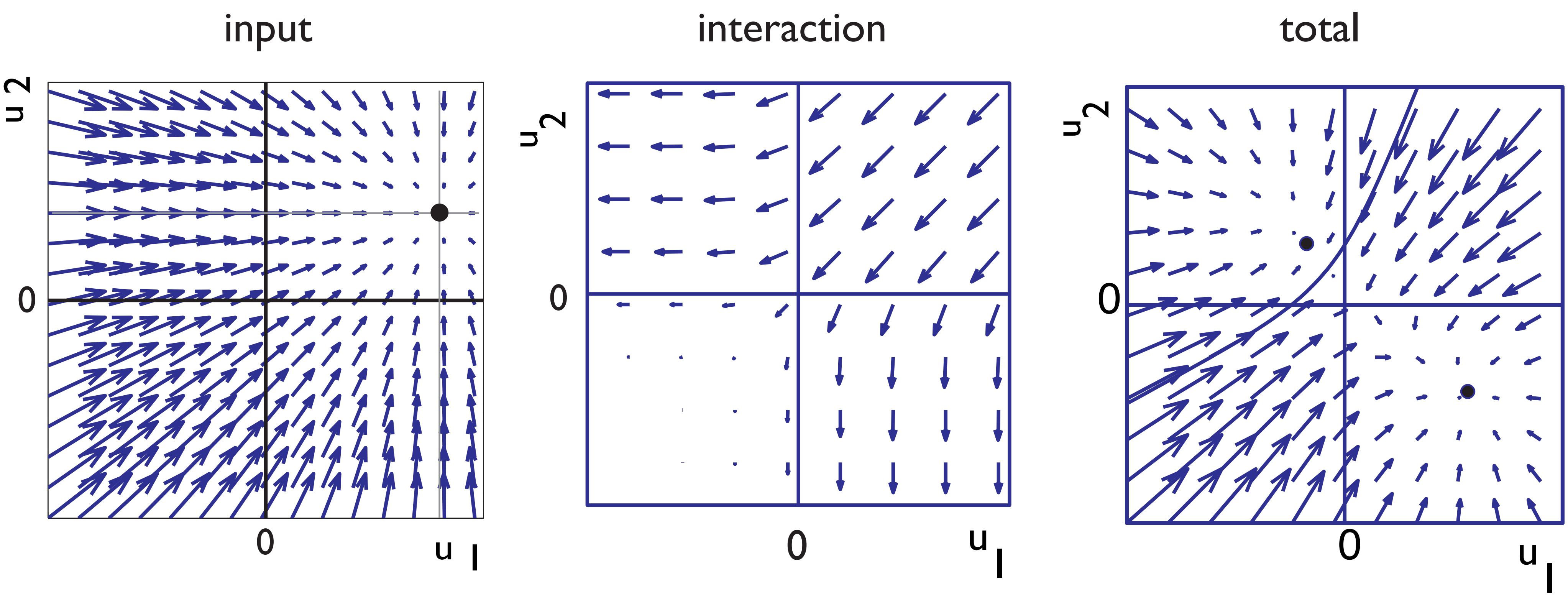


after input is presented



Neuronal dynamics with competition

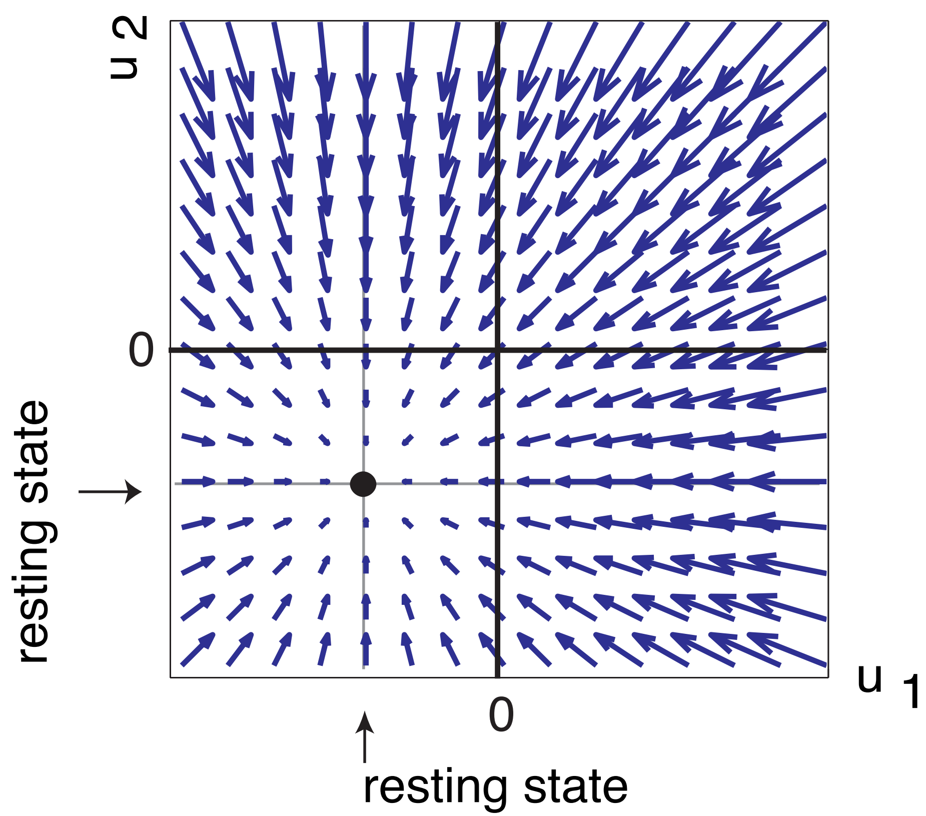
stronger input to u_1 => attractor with positive u_1 stronger, attractor with positive u_2 weaker => closer to instability



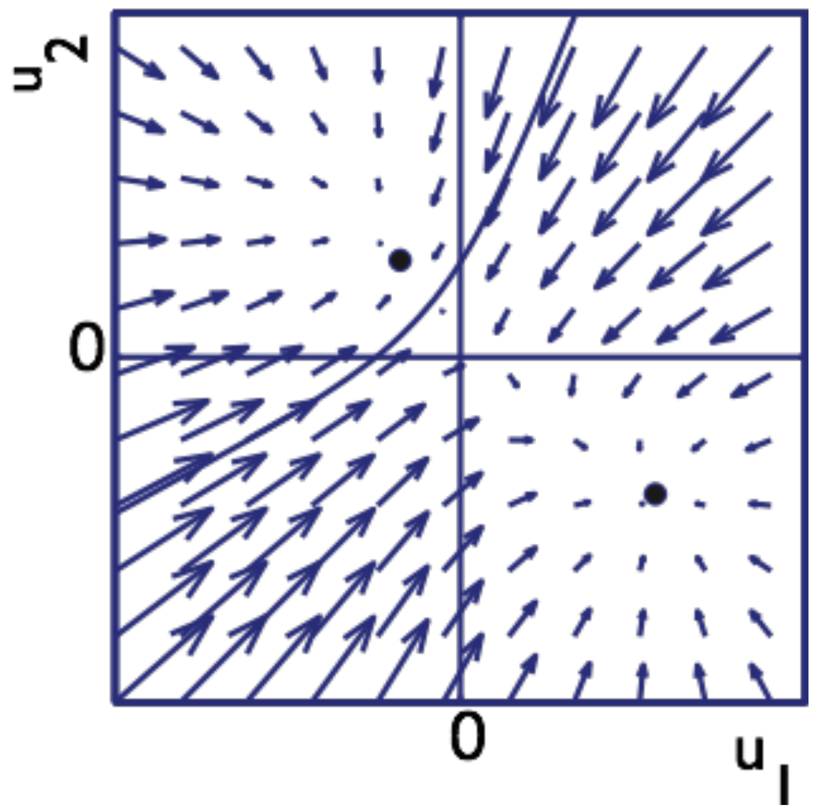
Neuronal dynamics with competition

- decision made at detection instability!

before input is presented



after input is presented

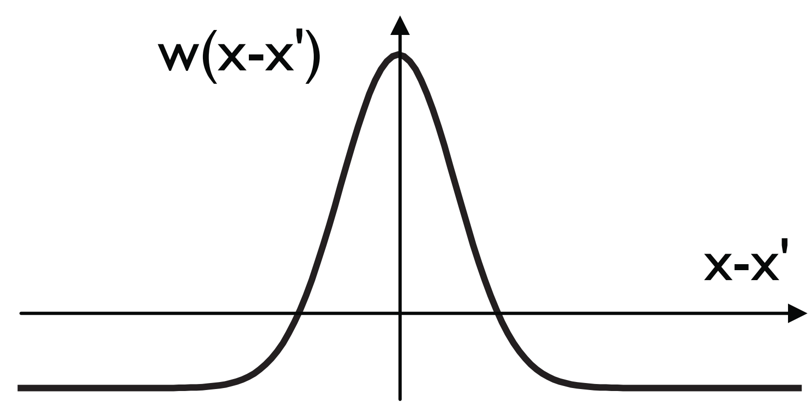


=> simulation

The neural dynamics of fields

- … the same underlying math
- coupling amoung continuously many activation variables
- local excitatory coupling ("self-excitation")
- global inhibitory coupling ("mutual inhibition")

$$\tau \dot{u}(x,t) = -u(x,t) + h + s(x,t) + \int dx' w(x-x') \; \sigma(u(x',t))$$



field vs. activation variables